Inversion Problem in Regional Electromagnetic Induction Studies

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Abstract - For completeness of the theory proposed by Oni (1972), the inversion problem is examined. It is shown that parameters which constrain distinct solutions of the inverse problem can be determined in the application of the theory. The fundamental basis of the inversion and the strategy to be adopted are discussed.

Key words: Electromagnetic induction; Inversion theory, electromagnetic.

1. Introduction

One of the most important basic problems in the interpretation of geophysical data has always been the non-uniqueness of earth models used to fit the experimental data. In some cases, this factor is either played down or ignored. In many cases, confirmation is sought from other geophysical techniques. But all of these techniques are in fact saddled with the problem of non-uniqueness. Hence one should not be too over-confident in the knowledge that one earth model has been confirmed by another geophysical technique when the interpretation of both sets of data is subject to the limitation imposed by the undefined non-uniqueness of the models.

Many authors, including BACKUS and GILBERT (1966), JACKSON (1973) have tackled the Inversion problem.

ANDERSSSEN (1970) examined the character of non-uniqueness in the conductivity modelling problem for a spherically symmetric earth, in order to show that methods for determining upper and lower bounds on the possible range of values for electrical conductivity as a function of depth, must be developed, before obtaining a better picture of conductivity within the earth. ANDERSSSEN (1974) gave an excellent review on the inversion of global electromagnetic induction data (GEMI).

The emphasis so far has been on the inversion problems of global electromagnetic induction data. Here the author wishes to look into the inversion problem in regional studies. WEIDELT (1974) considered the inversion of two-dimensional structures. He assumed that within a known normal conductivity structure, there is embedded an unknown laterally non-uniform, anomalous domain. He deduced the conductivity within the anomalous domain from a knowledge of the normal conductivity structure and the anomalous electromagnetic field, observed for various frequencies at the surface of the earth. The problem can be regarded as a local one because it is neither

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global nor regional in the sense used here. The inversion formulae were obtained in two cases where the anomaly is due to an undulation of a perfectly conducting interface and a lateral variation of the integrated conductivity of a thin sheet at $z = 0$. A quasi-uniform external magnetic field is assumed, SCHMUCKER (1970) worked out a simple relation which yields an approximate estimate of the conductivity variations with depth if the conductivity increases with depth.

$$C(\omega) = \frac{E_x(\sigma, \omega)}{i\omega\mu_0 H_z(\sigma, \omega)}$$

The Schmucker relation is suitable for regions of uniform source field, apart from the condition that the approximation works best if the conductivity increases with depth.

WEIDELT (1972) obtained an inversion formula for the case of a layered earth and uniform source field. The procedure is essentially based on the method of GEL'FAND and LEVITAN (1951a, b) for the solution of the inverse Sturm–Liouville problem. He also considered the case of an arbitrary external field and a flat earth. For a flat earth

$$E(r, \omega) = \int \int_{-\infty}^{\infty} a(v, \omega) w(z, v, \omega) \times \text{grad} \{\exp [i(v \cdot r + \omega t)]\} \, dv_x \, dv_y \, d\omega$$

where $E$ is the tangential solenoid vector for any solenoidal inducing field, $v = v_x \hat{x} + v_y \hat{y}$ is the horizontal wave vector, and $v = |v|$ is the wave number. $\hat{x}, \hat{y}, \hat{z}$ are unit vectors in $x, y, z$ directions. In solving the inverse problem for an arbitrary non-uniform source field, the inverse problem is reduced to that of a flat earth and a uniform source field by simple transformations. But it is assumed that $v$ is independent of frequency. This assumption simplifies the Inversion problem. But in the theory proposed by ONI (1972), no assumption on the variation of the source field function with frequency was made. Subsequently, in the application of the theory, it was found that the horizontal wave vector varies both with frequency at a single station and from station to station for the same frequency (ONI and AGUNLOYE 1973 and 1974).

Starting from the theory proposed by ONI (1972) and its subsequent application to electromagnetic induction study in regions of non-uniform source field, the fundamental basis of the inversion and a strategy for the solution of the inversion problem are discussed. This is essential for completeness of the theory.

2. The Inversion Problem

The following equations were derived by ONI (1972) for estimating the horizontal wave vector $v(\omega)$ of an arbitrary inducing source field