Two-layer Rotating Hydraulics: Strangulation, Remote and Virtual Controls

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*Abstract*—The hydraulics of two-layer, rotating channel flow is examined in the limit where the channel width is large compared to the internal Rossby radius of deformation, but small compared to the external deformation radius. In this limit the baroclinic flow is contained in boundary layers along each side wall, while the barotropic flow is distributed over the width of the channel. Width variations along the channel cause the strength of the barotropic flow to vary and the barotropic variations influence the baroclinic boundary layers in two independent ways. The dual nature of this forcing gives rise to a new type of critical condition which we refer to as a 'remote' control. 'Virtual' and 'narrow' controls also arise. Steady solutions can be obtained by solving a pair of simple quadratic equations and examples are given showing various combinations of controls.

**Key words:** Hydraulics, rotating hydraulics, remote control, virtual control, Fram strait, strangulation.

1. *Introduction*

Recent field experiments in Gibraltar and other sea straits have kindled a renaissance in two-layer hydraulics (e.g., Dalziel, 1988, 1990a,b; Armi and Farmer, 1986, 1987, 1988; Farmer and Armi, 1986). Although most studies concentrate on nonrotating flow, a few attempt to include rotation (Whitehead et al., 1974; Hogg, 1983; Dalziel, 1988, 1990a). As the sophistication of models grows, it becomes more difficult for the nonspecialist to understand the complicated flow regimes with their multiple control points. Whereas analytic descriptions of key physical processes can be made in classical, single-layer systems, more reliance on numerical solutions is made in the multi-layer, rotational systems. These technical difficulties make it harder to illustrate the general behavior of steady solutions, particularly the along-channel structure.

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In this paper we examine the hydraulics of a system containing both multiple density layers and rotation in an asymptotic limit allowing simple mathematical descriptions of physical processes. In particular, we consider two-layer flow through a contraction in a rotating channel of width much smaller than the external Rossby radius (rigid lid), but much greater than the internal Rossby radius. Although there are many sea straits where such a model might be applicable (the Drake Passage, Florida Straits, Fram Strait, Bering Strait, and others) no comparisons with observations will be made. The primary purpose is to present a calculation which will provide intuition into the forcing mechanisms, critical controls, and depth and velocity structures characteristic of hydraulically-driven, two-layer flow with rotation and to point out some new dynamical properties. In the process we identify some novel features, namely a new type of forcing mechanism, a new type of critical control, and a rich variety of steady solutions.

A general formulation of the two-layer, rotating, rigid-lid problem for flow over a sill and/or through a contraction has been made by Dalziel (1988, 1990a). Dalziel performs the calculation using a formalism introduced by Gill (1977), which requires specification of the flow at any given cross-section in terms of a single dependent variable. The algebra involved is formidable and Dalziel later simplifies the calculation by considering two special cases, the first having equal potential vorticities and equal and opposite flow rates in each layer (also considered by Whitehead et al., 1974 for zero potential vorticity) and the second having zero potential vorticity in each layer. The steady solutions for flow through a contraction and/or over an obstacle become analogous to those of nonrotating two-layer flows provided that the interface does not separate from either of the channel side walls. In particular, the influence of sills (as opposed to width contractions) on the flow rate, the placement of control sections, and the conditions for maximal exchange flow are all similar to the nonrotating analog. With separation, significant departures from nonrotating theory and even breakdown of the underlying approximations occur which are not yet completely understood. Hogg (1983) discusses some of these ideas in connection with a study of the deep flow in the Vema Channel. Similar difficulties exist in one-layer flows with rotation. In the present treatment, as in Dalziel (1988, 1990a), we will concentrate on the nonseparated case.

We believe that the special cases considered by Dalziel, though very instructive, eliminate some of the features which distinguish two-layer rotating flow from single-layer or nonrotational two-layer flow. These features stem from the peculiar forcing of the baroclinic portion of the flow, which can be described by reference to the strait shown in Figure 1. If the external Rossby radius of deformation is much greater than the width of the strait, the free surface of the flow is nearly horizontal. In addition, if the internal Rossby deformation radius is much smaller than the width, the slope of the interface between upper and lower layers can be significant. As will be shown, the interface can become banked against the side walls and the baroclinic part of the flow confined to boundary layers, as shown by the dashed lines...