Resistivity and Induced Polarization Response of a Thin Sheet

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Abstract—The relevant potential theory is given for a current point source in the presence of a conductive slab embedded in a homogeneous host region of infinite extent. The thin sheet representation is obtained from the exact integral formulation by a simple mathematical limit process. The same result is also deduced directly at the outset by applying a thin sheet boundary condition. The apparent resistivity for a two electrode array is then computed for the case where the bore hole intersects the thin sheet at right angles. The corresponding results for the dilution factor, relevant to the induced polarization response, are also obtained. It is shown that the apparent resistivity and the dilution factor are constant when the potential and the current electrode straddle the sheet but there is a characteristic decrease as the electrodes move away from the sheet.

Key words: thin sheet, induced polarization, resistivity, borehole, conductive slab, dilution factor.

Introduction

In this paper the use of the thin sheet approximation for resistivity probing is illustrated. In particular the model is relevant to the case where a two electrode system is being employed to measure the apparent resistivity in the rock formation adjacent to a borehole. However, in our idealized formulation the influence of the borehole itself is ignored. Such an assumption is justifiable if the borehole is air filled but of course some further correction is needed if the hole is filled with highly conducting fluid. (e.g. see WYLIE, 1957)

Basic formulation

In our particular case the electrode system will be a direct current source placed at some distance from a potential electrode (normal array). The geometry of this problem is shown in Figure 1. In this model, C is considered a current point source imbedded in a homogeneous medium of resistivity $\rho_1$ and $P$ is the potential electrode. The z-axis represents the borehole and a homogeneous conductive slab of thickness

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Figure 1
Geometry for three region borehole problem.

$t$ and resistivity $\rho_2$ is located at a depth $d$ below $C$. The integral expressions for the potentials $W_1$, $W_2$ and $W_3$ in the three regions will be obtained. The slab will then be considered a thin conductive sheet, a procedure that allows the thin sheet approximation to be employed in obtaining the potentials above and below the thin sheet. The integral expressions for the potentials will then be expressed in terms of the exponential integral and used to plot the apparent resistivity and the dilution factor. In the formulation the lower half space will have a resistivity $\rho_3$.

**Formulation for potentials**

The potentials for the three regions in Figure 1 can be found by following Wait (1982). Thus, for $z > -d$,

$$W_1 = W^p + W^s$$

$$= \frac{1}{4\pi R} \int_0^\infty A(\lambda)e^{-\lambda z}J_0(\lambda r)d\lambda$$

where

$W^p$ = the primary potential due to current source $C$,

$W^s$ = the secondary potential due to interface,

$A(\lambda)$ is an unknown function,

and

$$R = \sqrt{r^2 + z^2}$$