THREE-DIMENSIONAL VISCOS FLOWS IN FURNACE CHAMBERS

V. B. Dovzhik and V. K. Migai

A method is proposed for calculating isothermal flows in channels of complex configuration, for the example of a vortex furnace chamber.

Current furnace technology is distinguished by great diversity of aerodynamic schemes: As well as the traditional opposed grouping of vortex or direct-furnace burners and the tangential grouping, other types employed are the furnaces designed by the I. I. Polzunov Central Scientific-Research Institute for the Planning and Design of Boilers and Turbines (CSIPDBT) and the Leningrad Polytechnic Institute, cyclone furnaces, multifaceted annular furnaces, and furnaces with plane-flame burners. This diversity of aerodynamic schemes is not by chance, since aerodynamics is one of the factors determining the combustion and heat-transfer processes in furnaces. Therefore, in developing new designs of boiler units, the study of gas-flow aerodynamics inside furnace chambers is of great importance. To date, this question has basically been approached by numerical modeling. However, the development of methods of computational hydrodynamics and progress in computer technology permits the use of mathematical methods of modeling the aerodynamic processes together with physical methods. The promise of this approach is indisputable, since a successful mathematical model may permit operative change in the boundary conditions and configuration of the design, and subsequently the consideration of heat and mass transfer in the boiler units, together with aerodynamics.

However, mathematical modeling of furnace processes encounters large but not insuperable difficulties associated with the complexity of the phenomena being modeled and the imperfection of the computer technology employed (inadequate speed and memory capacity). Flow in furnace chambers is spatially complex, and thus the problems to be solved are unconditionally three-dimensional. Only certain forms of flow [1] may be described in a two-dimensional coordinate system, offering the possibility of including the model of turbulence and combustion in consideration. In solving three-dimensional problems, on the one hand, the range of problems considered must be restricted and, on the other, coarse grids must be used.

In the present work, in constructing a numerical method of investigation, it is taken into account that the maximum velocities in the furnaces are no greater than 100 m/sec. At temperatures of around 1800 K, this corresponds to a Mach number $\text{M} \approx 0.12$. At such values of $\text{M}$, the liquid may be regarded as incompressible.

In flame ignition of fuel and oxidizing agent, jets are introduced in the furnace volume and, interacting with one another and with the boundary surfaces, mix well and form a single flow; this allows the flow to be regarded as isothermal, in the first approximation.

Turbulent motion of the incompressible liquid may be described by the Reynolds equations. If they are closed using the Boussinesq hypothesis and the assumption that the turbulent viscosity is constant and much larger than the molecular viscosity, these equations reduce to the Navier–Stokes equations. As well as the continuity equation, the following system is obtained here

$$
\sum_{k=1}^{N} \left( \mu_h \frac{\partial u_i}{\partial x_k} - \frac{1}{Re_{ef}} \frac{\partial^2 u_i}{\partial x_k^2} \right) + \frac{\partial P}{\partial x_i} = 0, \; i = 1, ..., N, \; \sum_{k=1}^{N} \frac{\partial u_h}{\partial x_k} = 0.
$$

The assumption of constant turbulent viscosity is fairly rough. However, taking account of the lack of information on turbulent viscosity for such flows, the three-dimensionality of
the problem, and the complexity of the algorithm when even simple models of turbulence are used, this assumption is adequate for the first stage of the investigation. The calculations are performed in a wide range of Re_{ef}. The theoretical velocity fields best agreeing with experiment are noted below.

At the boundaries of the given region, conditions for the components of the velocity vector are specified; the pressure at an arbitrary point is assumed to be zero, and the pressure is determined accurately in the volume, except for some additive constant.

It is convenient to write Eq. (1) in vectorial form in order to describe the algorithm for solution of the given problem

\[
\begin{bmatrix}
- u_1 \\
\vdots \\
u_N \\
p
\end{bmatrix}
\begin{bmatrix}
L & 0 & \cdots & 0 \\
0 & \ddots & \ddots & 0 \\
0 & \cdots & 0 & L \\
0 & \cdots & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial x_1} \\
\vdots \\
\frac{\partial}{\partial x_N}
\end{bmatrix}
\]

The economic difference scheme constructed in [2, 3] on the basis of the fractional-step method is used. The solution is sought by the establishment method, setting the steady Navier-Stokes equations in correspondence with their nonsteady analogs, and the additional term \((1/A_p)(\partial P/\partial t)\) is introduced in the continuity equation

\[
A \frac{\partial f}{\partial t} + A f = 0, \quad f_{|t=0} = f_0, \quad A = \begin{bmatrix}
1 & \cdots & 0 & 0 \\
\vdots & \ddots & \vdots & 0 \\
0 & \cdots & 1 & 0 \\
0 & \cdots & 0 & \frac{1}{A_p}
\end{bmatrix}
\]

If \(\Lambda(s)\) is the difference matrix operator approximating the differential operator \(\Lambda\) with order \(s\) relative to the spatial step \(h\), the difference analog of Eq. (3) may be the system

\[
A \frac{f^{n+1} - f^n}{\tau} + \Lambda^{(n)} f^{n+1} = 0.
\]

If Eq. (4) is to be linear, the operator \(\Lambda(s)\) must be taken at the \(n\)-th layer, here and below. For simplicity of notation, the superscript \(n\) is omitted.

The values of the components of the vector \(f^{n+1}\) at time \(t^{n+1}\) are found from their values at time \(t^n\), i.e., \(f^{n+1} = f^n + \xi^{n+1}\). Then it follows from Eq. (4) that

\[
(E + \tau A^{-1} \Lambda^{(n)}) \xi = - \tau A^{-1} \Lambda^{(n)} f^n,
\]

where \(E\) is a unit matrix and

\[
\xi = \begin{bmatrix}
\xi_{u_1} \\
\vdots \\
\xi_{u_N} \\
\xi_{p}
\end{bmatrix}
\]

Resolving the operator \(\Lambda_N(s)\) with respect to the spatial variable: \(\Lambda^{(n)} = \sum_{j=1}^{N} \Lambda_j^{(n)}\), where \(\Lambda_j^{(s)}\) includes the approximations of all the derivatives with respect to \(x_j\), and implicit