Flow of a liquid carrying an electric current around a nonconducting drop is considered. The results are used for determining the rate of mass exchange between the drop and the surrounding liquid.

If a current-carrying liquid surrounds a body whose electrical conductivity is different from that of the liquid, the nonuniformities which arise in the density distribution of the electric current produce a nonpotential field of electromagnetic forces. These forces alter the character of the flow near the body and thereby affect the rate of mass exchange between the body and the current-carrying liquid.

Consider the low-velocity flow produced by electromagnetic forces near a stationary drop. Assuming that the drop retains its shape and neglecting the inertial terms, we write the equation of motion (the coordinate system is shown in Fig. 1),

\[ -\text{grad} P - \mu \text{rot rot} U \times \mathbf{B} = 0 \]  

or

\[ \mu \text{rot rot} U = \text{rot} (I \times \mathbf{B}). \]  

The expressions for \(I \times \mathbf{B}\) and \(\text{rot}(I \times \mathbf{B})\) in the induction-free approximation are given in [1]. They have the following form:

\[ I \times \mathbf{B} = \frac{-1}{2} \mu \sigma \frac{l_0}{r} \sin \theta \left[ 1 - \left( \frac{a}{r} \right)^3 \right] \left\{ i \frac{1}{2} \sin \theta \left[ 2 + \left( \frac{a}{r} \right)^3 \right] + i \frac{1}{2} \cos \theta \left[ 1 - \left( \frac{a}{r} \right)^3 \right] \right\}; \]  

\[ \text{rot}(I \times \mathbf{B}) = -i \frac{3}{2} \mu \sigma \frac{l_0}{r} \sin \theta \cos \theta \left( \frac{a}{r} \right)^3 \left[ 1 - \left( \frac{a}{r} \right)^3 \right]. \]  

We introduce a stream function \(\psi\), such that

\[ u_r = -\frac{1}{r^2 \sin \theta} \cdot \frac{\partial \psi}{\partial \theta}; \quad u_\theta = -\frac{1}{r \sin \theta} \cdot \frac{\partial \psi}{\partial r}. \]  

By substituting (4) and (5) in (2), we obtain the equation determining the stream function for the region outside the drop:

\[ \left[ \frac{\partial^2}{\partial r^2} + \sin \theta \cdot \frac{\partial}{\partial \theta} \left( \frac{1}{r \sin \theta} \cdot \frac{\partial}{\partial \theta} \right) \right] \psi = -\frac{3}{2} \frac{\mu \sigma}{\mu \sigma} \frac{l_0}{r} \sin^2 \theta \cos \theta \left( \frac{a}{r} \right)^3 \left[ 1 - \left( \frac{a}{r} \right)^3 \right]. \]  

For the region inside the drop, we correspondingly have

\[ \left[ \frac{\partial^2}{\partial r^2} + \sin \theta \cdot \frac{\partial}{\partial \theta} \left( \frac{1}{r \sin \theta} \cdot \frac{\partial}{\partial \theta} \right) \right] \psi' = 0. \]

Before solving Eqs. (6) and (7), we must determine the form of the supposed solution. This can be done by investigating the character...
of the electromagnetic forces causing the motion of the liquid. It is evident from Eq. (3) that the field of electromagnetic forces is axisymmetric with respect to the X axis and symmetric with respect to the YOZ plane. Since the liquid flow possesses a similar symmetry, the stream function should be sought in the following form:

$$\psi = R(r) \sin^2 \theta \cos \theta.$$  \hfill (8)

By substituting (8) in (6) and assuming that $$R(r) = \text{const} \, r^n$$, we obtain

$$\psi = - \frac{\mu_e}{16\mu} \left[ \frac{a}{r} + \left( \frac{r}{a} \right)^2 + C_1 \left( \frac{a}{r} \right)^2 + C_2 \right] \sin^2 \theta \cos \theta$$  \hfill (9)

outside the drop, and

$$\psi' = \left[ C_1 \left( \frac{r}{a} \right)^3 + C_2 \left( \frac{r}{a} \right)^5 \right] \sin^2 \theta \cos \theta$$  \hfill (10)

inside the drop. It has been taken into account here that the velocity must be bounded. For determining the constants, we have for $$r = a$$

$$u'_a = u_0; \quad \psi' = \psi = 0;$$

$$\mu' \left( \frac{1}{r} \cdot \frac{\partial u'_a}{\partial \theta} - \frac{\partial u'_a}{\partial r} - \frac{u'_a}{r} \right) = \mu \left( \frac{1}{r} \cdot \frac{\partial u_0}{\partial \theta} - \frac{\partial u_0}{\partial r} - \frac{u_0}{r} \right).$$  \hfill (11)

By using conditions (11), we obtain for the stream function outside the drop

$$\psi = - \frac{\mu_e \rho_a a^3}{16\mu} \left[ \frac{a}{r} + \left( \frac{r}{a} \right)^2 \frac{5\beta - 2}{10(\beta + 1)} + \frac{25\beta + 18}{10(\beta + 1)} \right] \sin^2 \theta \cos \theta$$

and for the stream function inside the drop

$$\psi' = \frac{7}{160} \cdot \frac{\mu_e \rho_a a^3}{\mu} \cdot \frac{1}{1 + \beta} \left[ \left( \frac{r}{a} \right)^3 - \left( \frac{a}{r} \right)^5 \right] \sin^2 \theta \cos \theta,$$

where $$\beta = \mu' / \mu$$.

If $$\beta \to 0$$ (solid sphere), then

$$\psi = - \frac{\mu_e \rho_a a^3}{32\mu} \left[ 2 \left( \frac{a}{r} \right)^2 + 2 \left( \frac{r}{a} \right)^2 + \left( \frac{a}{r} \right)^2 - 5 \right] \sin^2 \theta \cos \theta,$$

which coincides with the result obtained in [1].

If $$\beta \to 0$$ (gas bubble),

$$\psi = - \frac{\mu_e \rho_a a^3}{16\mu} \left[ \left( \frac{a}{r} \right)^2 - \left( \frac{r}{a} \right)^2 - \frac{1}{5} \left( \frac{a}{r} \right)^2 - \frac{9}{5} \right] \sin^2 \theta \cos \theta.$$