thickness of the frost on the initial section, at a distance $l$, and the average thickness of the layer; $\delta^* = \delta_0 l^{-1}; L = \frac{L_0}{l}$. 

LITERATURE CITED


FILTRATION OF A MAGNETIC FLUID IN A DEFORMABLE POROUS MEDIUM

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The equations of motion of a magnetizing fluid are obtained in a deformable non-magnetic porous medium.

Filtration of a magnetic fluid in nondeformable porous media was examined in [1, 2]. Derivation of the equations of magnetic fluid filtration in a deformable porous matrix consisting of deformable grains that are displaceable relative to each other is of interest.

It is assumed that an inhomogeneous magnetizing fluid fills the pore space entirely, i.e., the medium is saturated; there are no phase transitions associated with absorption (desorption) of the solid ferromagnet particles on the pore surface. The equations of fluid motion in a porous medium are obtained by local volume averaging [3] of the microequations of fluid motion in the pores, the Maxwell equation for the magnetic field in the pores and the matrix, as well as the equations of porous matrix deformation, with thermal expansion of the grains, from which the matrix consists, and the relative grain displacement taken into account. The magnetic properties of the medium as a whole (matrix + fluid) are characterized by the effective magnetic permittivity of the medium. The interphasal heat transfer between the liquid and solid phases is taken into account in the averaged heat conduction equations for the fluid and porous matrix.

The following relationships [3]

$$\langle \nabla \hat{I}_a \rangle = \nabla \langle \hat{I}_a \rangle + \sigma_{12} \langle n_{a1} \hat{I}_a \rangle_{12},$$

$$\langle \partial_t \hat{I}_a \rangle = \partial_t \langle \hat{I}_a \rangle - \sigma_{12} \langle n_{a1} u^i \hat{I}_a \rangle_{12},$$

are used to average the microequations.


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The microequations of the fluid motion in the pores, the deformation equations of the porous matrix, and also the boundary conditions on the interphasal surface are written in the form [4-6]:

\[
\begin{align*}
\partial_t \rho_a v_{ai} &= -\nabla_k \rho_a v_{ai} v_{ak}^k + \nabla_h \rho_{ai} + \rho_a g_i, \\
\partial_t \rho_a + \nabla_h \rho_a v_{ak}^k &= 0, \quad \nabla_h B_{ak}^k = 0, \quad \epsilon_{ijk} \nabla_j H_{ah} = 0,
\end{align*}
\]

\[
B_{ah} = H_{ah} + 4\pi M_{ah},
\]

\[
p_{ih} = -\rho_i g_{ih} + \frac{1}{4\pi} H_{ih} B_{ih} + \tau_{ih},
\]

\[
p_i = \rho_{ai} + H_i^2 + \frac{H_{ih}}{8\pi} \left[ M - \rho \left( \frac{\partial M}{\partial \rho} \right)_{T,H} \right] dH,
\]

\[
\tau_{ih} = 2\eta_i \epsilon_{ih}, \quad \epsilon_{ih} = (1/2) (\nabla_i v_{ih} + \nabla_h v_{ih}),
\]

\[
\rho_a c_a \frac{dT_a}{dt} = \nabla_h \kappa_a \nabla h T_a,
\]

\[
p_{ih} = \lambda_a g_{ih} \epsilon_{ih}^{a2} + 2\mu_a \epsilon_{ih} + \beta_{2T} K_2 g_{ih} (T_2 - T_0) + \frac{1}{4\pi} \left( H_{gi} H_{ih} - \frac{1}{2} H_{gih}^2 \right),
\]

\[
\epsilon_{ih} = (1/2) (\nabla_i h_{2h} + \nabla_h h_{2h}), \quad v_{ih} = \frac{d_a h_{2j}}{dt},
\]

\[
\{B_i\} = 0, \quad \{H_{ih}\} = 0, \quad \{\epsilon_{ih}\} = 0,
\]

\[
\{\rho_{ih} \} = 0, \quad \{T\} = 0, \quad \{\kappa T\} = 0.
\]

The equations in which the quantities are denoted with the subscript 1 refer to the liquid phase, and with the subscript 2 to the solid phase, and with the subscript a (α = 1; 2) to both phases. Since the matrix is assumed nonmagnetic, we should set \( M_{2h} = 0 \). The difference in the specific heats for constant pressure and volume is not taken into account in (2) and henceforth. Summation is over repeated subscripts. We neglect the magnetocaloric effect as well as the work of the internal forces in the heat influx equations for the liquid and solid phases. Since the deformations within the grain (but not the relative displacements of the grain) are small, we neglect the convective term in the substantial derivative (i.e., \( \partial_a / \partial t \approx \partial / \partial t \)) as well as the first term in the right side of the momentum equation for the solid phase, which is the convective momentum transport. The equations of state of both phases should also be appended to (2).

Taking the average, with respect to the phases, according to (1), of the momentum equations for both phases (2) and combining them, with the boundary conditions (2) taken into account, we find the momentum equation for the mixture (matrix + fluid)

\[
\partial_t (m_1 <\rho_1>_1 <v_{ih}>_1 + m_2 <\rho_2>_2 <v_{ih}>_2) = -\nabla_h m_1 <\rho_1>_1 <v_{ih}>_1 <v_{ih}^k>_1 - \nabla_h m_1 <\rho_1>_1 <v_{ih}^k>_1 + \nabla_h (m_1 <\rho_1>_1 + m_2 <\rho_2>_2) + g_i (m_1 <\rho_1>_1 + m_2 <\rho_2>_2).
\]

(3)

Here \( \varphi_i = v_i - <v_i>_1 \) is the fluctuation of the quantity, i.e., its deviation from the mean value. Furthermore, we neglect the second term in the right side of (3), which is the fluctuating momentum transport.

We write the averaged momentum equation for the liquid phase in the form

\[
\partial_t m_1 <\rho_1>_1 <v_{ih}>_1 = -\nabla_h m_1 <\rho_1>_1 <v_{ih}>_1 <v_{ih}^k>_1 + R_{21i} + \nabla \sigma_{ih}^k + m_1 <\rho_1>_1 g_i,
\]

\[
\sigma_{1ih} = m_1 <\rho_{ih}>_1, \quad R_{21i} = \sigma_{1ih} <\rho_{ih}^k>_1.
\]

Here \( R_{21i} \) is a vector governing the force effect of the phase 2 on the phase 1 per unit volume of mixture; the tensor \( \sigma_{1ih} \) determines the stress acting on phase 1 on the surface of a certain mixture volume.