The Effective Age of Bubbles in Polar Ice

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Abstract—A mathematical description of the trapping of air bubbles in polar ice is analysed in order to assist in the interpretation of measurements of anthropogenic constituents which have recently increased on time scales comparable to the firn closure time. The effective age of a layer of ice is defined in terms of the time at which the atmospheric concentration of a constituent was equal to the mean concentration for bubbles found in that layer. Under the assumption of uniform snow deposition at a particular site, the effective age is found to be the same for all constituents that vary linearly throughout the trapping period for a layer. Using a trapping distribution based on theoretical and observational studies, the corrections for non-linearity are found to be small for typical anthropogenic constituents.

This property makes it possible to use smoothly increasing tracers such as the chlorofluorcarbons to determine the effective age empirically, even though it is an extremely poorly-conditioned problem to determine the entire trapping time distribution function by inversion of tracer concentrations.

Key words: Firn closure; atmospheric composition; bubble trapping.

1. Introduction

Recently there have been a number of analyses of the composition of air bubbles in polar ice for the purpose of reconstructing a history of changes in atmospheric composition over the last few centuries. Neftel et al. (1985) measured the CO\textsubscript{2} concentration in a sequence of ice layers and estimated that these concentrations corresponded to various times between 1750 and the present. Raynaud and Barnola (1985) measured CO\textsubscript{2} concentrations in 8 layers spanning a somewhat longer period, estimated at between 450 and 600 years, the uncertainty being due to limited information concerning the ice-core.

For a long time it has been realised that a gas sample from a particular ice layer will represent a mixture of atmospheric compositions over a range of times. Lorius and Raynaud (1983) believed that this would preclude the possibility of resolving relatively rapid changes in atmospheric composition. However Enting (1985a) suggested that if the trapping distribution can be determined, then the inversion problem of recovering an atmospheric composition history from a set of concentrations in ice layers is more stable than most other inversion problems occurring in geochemical modelling. The reason for this is that the trapping distribution function

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is sharply peaked as can be seen, experimentally, from the measurements of trapped bubble volume (Schwander and Stauffer, 1984), and theoretically, by modelling trapping using the percolation model from theoretical physics (Stauffer et al., 1985; Enting, 1985a).

While the full deconvolution of concentrations measured in polar ice is mathematically feasible, the CO₂ studies mentioned above simply assigned estimated gas ages to each of the measurements as representing atmospheric composition at the time indicated by these effective ages. The aim of this note is to determine the circumstances under which the concept of an effective gas age will be useful, thereby making a full deconvolution unnecessary.

The trapping of gas can be expressed in terms of a function \( f(z, z') \) which, for ice \( z \) years old, gives the amount of gas trapped \( z' \) years ago. The analysis in this paper assumes a constant deposition rate at any one site so that the trapping is a function only of the age of the ice at the time of trapping, i.e.,

\[
  f(z, z') = R(z - z').
\]

The function \( R \) will differ greatly for different sites. Because of this, it is desirable to be able to determine the important characteristics of the trapping as simply as possible since the process will have to be repeated for each core. We show that, under the assumption of constant deposition, the concept of an effective age is useful. For tracers with a linear variation over the trapping period, the effective age differs from the true age by a constant and the corrections for non-linearity are small. The results are mainly applicable to analysis of anthropogenic increases in atmospheric constituents over the last 150 years. On time scales of centuries to millenia, the assumption of constant deposition rate is unlikely to hold for any ice cores. For more recent times the degree of uniformity can be assessed from physical examination of the ice core.

2. The effective age of gas in ice layers

In the notation of Enting (1985a) the concentration, \( q(z) \), of a tracer in an ice layer of age \( z \) is related to the atmospheric concentration \( c(z') \) \( z' \) years ago by

\[
  q(z) = \int_{z}^{0} R(z - z') c(z') dz'.
\]

The variables \( z \) and \( z' \) are negative time co-ordinates that increase with depth in a core; they have their origins at the time that the core was extracted. The ‘dot’ notation, as in \( \dot{c} \), is used to denote derivatives with respect to \( z \).

The relationships between the various quantities are shown schematically in Figure 1. These relationships have been emphasised in the diagram by constructing \( q(z) \) from \( c(z) \) using a function \( R(z) \) that has a realistic cutoff at \( z_0 \), but which has a broader peak than seems plausible. Consider the layer of age \( z_1 \). The lowest curve gives \( c(z_1) \), the atmospheric concentration when the layer was deposited. The upper