Love Wave Dispersion:
Errors due to Assumption of Isotropy

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Summary – Frequency equation for Love waves, propagating in a transversely isotropic homogeneous layer which is embedded between two isotropic homogeneous half spaces, is obtained. Errors, in rigidity ($\mu_H$), density and thickness of the anisotropic layer, which arise due to the assumption of isotropy of the medium are calculated by numerically analyzing the frequency equation. The results show that the errors increase with the increasing frequencies, and their signs depend upon the value (whether less than or greater than 1) of the anisotropy co-efficient of the medium of the layer.

1. Introduction

In a previous paper (NEGI and UPADHYAY [1] 3), the effect of anisotropy on Love wave propagation was studied in a simple model structure. It was shown that disregarding the anisotropy of the medium leads to an ambiguity in the interpretation of modes, since the same dispersion curve may correspond to two different mode numbers in isotropic and anisotropic medium respectively. Similar effects due to anisotropy may be expected to arise in the observational seismic studies, which if not taken into account, would give rise to erroneous structural interpretations. A pertinent problem for study is one in which the interpretations are based assuming isotropy whereas, in fact, the medium is anisotropic. In the following, analysis is presented for this type of situation considering fundamental mode of Love wave propagation.

2. Formulation of the problem

Figure 1 shows the layered configuration, direction of the co-ordinate axes and the elastic parameters of each medium. Propagation of Love waves is considered in the direction of positive x-axis in the anisotropic layer.

For Love waves propagating in the direction of x-axis, the displacement is in the direction of y-axis and is a function of x, z. The equation of motion in the anisotropic
Figure 1
Homogeneous transversely isotropic layer, lying between two isotropic homogeneous halfspace

layer is (Anderson [2], equation 1)
\[ \rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[ \mu_H \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \mu_v \frac{\partial u}{\partial z} \right] \] (1)

where \( \mu_H \) and \( \mu_v \) are directional rigidities, and \( u \) is the displacement. Corresponding equation for an isotropic (\( \mu_H = \mu_v = \mu \)) medium is
\[ \rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[ \mu \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \mu \frac{\partial u}{\partial z} \right]. \] (2)

Considering plane wave solution in the form
\[ u = V(z) e^{i(\omega t - kx)} \] (3)

and substituting it in equations (1) and (2) respectively gives
\[ \frac{d^2 V}{dz^2} - S_a^2 V = 0 \quad \text{(anisotropic)} \] (4)

and
\[ \frac{d^2 V}{dz^2} - S_i^2 V = 0 \quad \text{(isotropic)} \] (5)

where
\[ S_a^2 = k^2 \left( \frac{\mu_H}{\mu_v} \right) \left\{ 1 - \frac{c^2}{\beta_H^2} \right\} \]
\[ S_i^2 = k^2 \left\{ 1 - \frac{c^2}{\beta_i^2} \right\} \]
\[ \beta_H = \left\{ \frac{\mu_H}{\rho} \right\}^{1/2} \]