We consider rectilinear steady motion in uniflow hydroreaction systems. From the conservation laws one can derive, for the general case, an equation relating the drag force $F$ in a self-propelled system to the energy parameters which characterize a reactive jet: the total energy $\alpha$ supplied per-unit mass of propellant, the flow rate $\beta$, and the thermal power $q$. The problem has never before been treated in this formulation. The results may be useful for the design of respective uniflow systems and, especially, for an evaluation of their efficiency.

The general velocity diagram far ahead of and behind a given vehicle in a jet model is shown in Fig. 1. No specific type of engine is indicated here, because such diagrams are applicable to intake at the nose cone, as well as from the boundary layer. The only difference is that in the first case the flow rate $\beta_0$ will be the same in the boundary layer and in the trail, while in the second case the flow rates will be $\beta_0$ and $\beta_0 - \beta$, respectively.

For a variable-composition system with simultaneous attachment and separation at flow rates $m_c$ and $m_u$, there has already been derived a closed system of equations in [2], where all parameters are functions of time $t$. If the external forces are drag forces and if the medium itself is stationary, then we have the following system of equations projected on the direction of travel in a system of coordinates referred to the medium:

$$
\begin{align*}
\dot{m} &= m_c - m_u, \\
\dot{m}_u &= \int \mu m_u \, d\tau = -\int F \, d\tau, \\
\frac{m_0^2}{2} + \int \frac{u^2 m_u}{2} \, d\tau + Q + \int F \, d\tau &= E.
\end{align*}
$$

Here $m$ and $v$ are the mass and the velocity of the vehicle, $u$ is the effective absolute velocity of separation, $F$ is the modulus of the principal vector of external forces, $E$ is the total energy supply in the engine, and $Q$ is the heat loss in the reactive jet.

If the motion of the vehicle is steady, then the mass of the vehicle remains constant ($m = m_0$), and the first equation of system (1) yields

$$
\dot{m}_c = \dot{m}_u = 0.
$$

Moreover, obviously, the heat loss and the thermal power in the reactive jet are related as follows:

$$
Q = q t.
$$
Finally, if an energy $\alpha$ is supplied per unit mass of propellant, we have

$$\dot{E} = \alpha m_a = a\beta.$$  \hspace{1cm} (4)

We insert relations (2), (3), (4) into the second and the third equation of system (1), after which we differentiate these equations with respect to time. We account here for the fact that $\dot{v} = 0$ in a steady flow. Then

$$u\beta = F; \quad \frac{u^2\beta}{2} + Fv + q = a\beta.$$ \hspace{1cm} (5)

Eliminating the function $u$ from system (5) yields

$$\frac{F^2}{2\beta} + Fv = a\beta - q.$$ \hspace{1cm} (6)

The resulting equation relates the drag of the vehicle, when self-propelled, to the characteristics of the reactive jet, and it describes the balance of reactive power. Indeed, the right-hand side of this equation is equal to the reactive power, while its left-hand side is equal to the sum the power lost at the exhaust,

$$\frac{F^2}{2\beta} - \frac{u^2\beta}{2} = \beta \frac{(w - v)^2}{2}$$

and the thrust power $Fv$, i.e., the power expended on overcoming the drag.

It has thus been shown that in all cases the drag, calculated from the trail parameters beyond the reactive jet, is the dynamic characteristic which, regardless of the structural features of a uniflow reaction system, determines the self-propelling mode of travel. With another quantity serving as the dynamic characteristic, the general conservation laws (1) will not be valid.

Considering that $u = w - v$, we may write the first equation in (5) as follows:

$$F = \beta (w - v).$$ \hspace{1cm} (7)

This means that the quantity $\beta (w - v)$ must in all cases be regarded as the thrust in a uniflow reaction system, regardless of the structural features of the latter. This has also been demonstrated experimentally for the particular case of an engine operating in the trail, as shown in the monograph [1].

Let us now consider two special cases.

1. A Hydroreaction System with Propellant Intake at the Nose Cone. The flow rate is here the same in the boundary layer and in the trail so that, consequently, the drag in the hydroreaction system is equal to the tow drag. Inserting the expression $F = c_X (v^3/2)$ into Eq. (6), we obtain

$$a^2v^4 + 2a\beta v^3 - 2\beta (a\beta - q) = 0,$$ \hspace{1cm} (8)

where $\alpha = (c_X/2)$ is half the drag coefficient.

Equation (8) is a fourth-degree algebraic equation. The problem of determining the travel velocity can be solved uniquely if this equation has a single positive solution at all feasible values of its coefficients. From the necessary condition for motion $E > Q$ follows $(a\beta - q) > 0$. Then Eq. (8), after being rewritten as

$$a^2v^2 + 2a\beta = \frac{2\beta (a\beta - q)}{v^3},$$ \hspace{1cm} (9)

will indicate that the straight line $a^2v + 2a\beta$ and the cubic hyperbola $2\beta (a\beta - q)/v^3$ intersect and do so necessarily at only one point for all positive values of $v$.

Writing this equation in the form (9) is convenient for a graphical solution. The advantage of this proposed method is that one can calculate the travel velocity without knowing the kinetic characteristics of the reactive jet.