ON EPISTEMIC LOGIC

Wolfgang Lenzen's *Belief, Knowledge, and Probability: Systems of Epistemic Logic* is the first comprehensive and systematic work on epistemic logic since the appearance of Jaakko Hintikka's booklet *Knowledge and Belief* twenty years ago [5]. It is an intriguing inquiry into the foundations and problems of that subject, and aims at axiomatizing the logic of sentences which deal with episteme and doxa. Unlike Hintikka's pioneering but rather intuitive, unsystematic and semi-formalized exposition, Lenzen's book is a technical one and offers several interesting epistemic and doxastic calculi. In the following review, I shall first outline and evaluate Lenzen's program, and then suggest an alternative approach. The interested philosopher of medicine may determine whether epistemic logic might be of some importance to the philosophy and methodology of clinical judgment.

1. CLASSICAL EPISTEMIC LOGIC

The systems of epistemic logic discussed until the present time by epistemic logicians and axiomatized in Lenzen's book can be viewed as classical ones in the following sense. The basic logic they use is the classical propositional or first-order predicate logic which is merely supplemented by epistemic and doxastic predicates and corresponding axioms and rules of inference.

1.1. Lenzen's Pre-Axiomatic Studies

Lenzen's book consists of eight chapters. The first five chapters, 136 pages, comprise pre-axiomatic, more or less intuitive, introductory discussions. In the remaining three chapters consisting of 202 pages the author develops his systems of epistemic logic. The former will be briefly reviewed in this section. The latter are discussed in the next section.

In Chapter 1, Lenzen gives a brief historical account, and sketches the subject and aims of epistemic logic (pp. 1–8). According to him, the first aim of epistemic logic is (i) to systematize the multitude of epistemic terms by which we express our propositional attitudes towards states of affairs, e.g., 'I know that the Eiffel tower is in Paris', and (ii) to select the fundamental epistemic concepts; and its second aim is to develop 'logical' relations between and 'logical' laws for these concepts. His interpretation of the qualifying adjective 'logical' is completely in the tradition of classical formal logic and semantics. 'The logical validity of an epistemic principle amounts to its analytical validity' (p. 6).

A variety of epistemic terms is systematized in Chapter 2 by intuitively discussing their meanings, use, and semantic interconnections (pp. 9–18). Four terms are selected from among this variety as the fundamental epistemic concepts, the properly epistemic notion of 'knowing that' and the doxastic notions of 'being convinced that', 'believing that', and 'considering it possible that'. Let $x$ be any individual and $\alpha$ any proposition. Lenzen's syntactic use of the four terms is this: '$x$ knows that $\alpha$', '$x$ is convinced that $\alpha$', '$x$ believes that $\alpha$', and '$x$ considers it possible that $\alpha$', henceforth abbreviated, respectively, by $K(x, \alpha)$.
C(x, a), B(x, a), and P(x, a). For example, the doxastic statement 'David believes that
Manuel has whooping-cough' is of the structure B(d, WM) where 'd' denotes David and
'WM' is the statement 'Manuel has whooping-cough'. Lenzen's book is about the 'logic' of
sentences which contain at least one epistemic sentence of the form K(x, a), C(x, a),
B(x, a), or P(x, a) where a is any simple or complex proposition.

Chapter 3 is devoted to the discussion of 'intuitive fundamentals of epistemic proposi-
tional logic' (pp. 19–80). The author's aim is to convince the reader that there are (i)
epistemic sentences on the one hand which are analytical, i.e., 'logically valid epistemic
principles', and should therefore be preserved by an axiomatization; and (ii) epistemic
sentences on the other hand which are invalid and should therefore be excluded. Among the
first class, for example, are the following simple 'principles': 1

1) K(x, a) → K(x, K(x, a))
2) C(x, a) → C(x, C(x, a))
3) B(x, B(x, a)) → a
4) P(x, a) → C(x, P(x, a))
5) P(x, a) ↔ ¬C(x, ¬a)
6) P(x, P(x, a)) → P(x, a)

Lenzen establishes an epistemic-logical 'principle' of this kind by arguing that it is either an
analytical sentence or implied by other analytical sentences. For example, (4) is viewed to
be an epistemic principle 'beyond all question because it would be absurd to maintain of
someone that he considers it possible that a but doubts whether he does so' (pp. 28–29).
And, according to Lenzen, (6) is a logical consequence of (2) and (5). The proof he gives
amounts to this: C(x, ¬a) → C(x, C(x, ¬a)) is an instance of (2); by contraposition, it
implies ¬C(x, ¬a) → ¬C(x, ¬a); from this and (5) it follows P(x, ¬C(x, ¬a)) → ¬C(x, ¬a), and then (6). This simple example shows how classical-logical procedures are
imported into the program.

On several occasions one doubts whether Lenzen's appeal to the fundamental or derived
analyticity is convincing. For example, I am certain that the 'principles' (3) and (6) above
are not analytical epistemic sentences in my language. Sentence (3) says 'I believe that what
I believe is true'. However, I seldom have such an absurd belief. According to (6), 'if I
consider it possible that I consider it possible that ¬a then I consider it possible that a'. I
doubt seriously whether my epistemic behavior is in accordance with this 'principle'. In
Lenzen's opinion, such doubts are due to the circumstance that 'our linguistic competency
does not suffice for a reliable judgment about structures of the kind P(x, P(x, a))' (cf.
p. 29). This implausible position frequently occurring in the book evokes the following
question. Do Lenzen's principles belong to the realm of pragmatic linguistics, and thus to
psychology or sociology, or are they normative rules of epistemic rationality in the sense
that they say, e.g., if it is true of an individual x that P(x, P(x, a)) then he should P(x, a)?
In the first case, Lenzen's epistemic logic would not be a 'logic' at all but an imaginary
pragmatic linguistics developed in the logician's ivory tower; and in the second case, the
following question would arise. Why should we accept just this particular epistemic logic
and not another one? Unfortunately, such fundamental problems are not touched in the
book.

In the same chapter, Lenzen compares also the modal structures of his principles with
alethic-modal structures and shows that there are some isomorphisms between them. For
instance, the principle (1) above is isomorphic to the alethic-modal axiom □a → □□a where
□ is the modality of being necessarily true. By inter-systems comparisons of this kind the
author arrives at the hypothesis that the alethic-modal system S4.2 yields 'the' logic of
knowledge if □ is interpreted as K (p. 80).

Some interesting relations between the theories of comparative and quantitative subjective
probability on the one hand and the doxastic predicates C, B, and P on the other are