A number of studies, for example of D. M. Klimov [3, 4], D. S. Pel'por [8], V. A. Pavlov [6] and others, have been devoted to the investigation of problems of stability of the motion of a gyroscope on a movable base and to the computation of its drifts in the nonlinear formulation.

In the present work we study the effect of vibrations of the base on the stability of the equilibrium position of heavy and astatic gyroscopes in resonance conditions. The mechanisms and the conditions of stability of the motion of these gyroscopes are determined for subharmonic and combination resonances.

§1. Formalization of the Problem and Coordinate Systems. We consider a symmetric gyroscope in Cardan suspension with horizontal axis of rotation of the outer ring; the center of gravity of the rotor is displaced along the axis of the intrinsic rotation of the rotor in relation to the fixed point of the gyroscope. The centers of inertia of the Cardan suspension rings coincide with the fixed point of the gyroscope (Fig. 1).

The base of the gyroscope has an arbitrary motion in space. It is assumed that the mass of the base of the gyroscope is much greater than the mass of the Cardan suspension rings and the rotor taken together; therefore the effect of the motion of the latter on the motion of the base can be disregarded.

We introduce the following systems of coordinates: $0\xi\eta\zeta$ is a coordinate system having a translational motion relative to the inertial system; systems $0\xi y_1 z_1$, $0\xi y_2 z_2$, $0\xi y_3 z_3$ are fixed to the base, the outer ring, the case, and the rotor of the gyroscope respectively. The point $0$ coincides with the center of Cardan suspension. The axis $0x_1$ is directed along the axis of rotation of the outer ring, $0y_2$ along the axis of rotation of the case, and $0z_3$ along the axis of rotation of the rotor. We shall take the axes of the coordinate systems $0\xi y_1 z_1$, $0\xi y_2 z_2$, $0\xi y_3 z_3$ to be the principal axis of inertia of the bodies to which they are fixed. At the initial instant of time the corresponding axes of the system $0\xi\eta\zeta$, $0\xi y z$, $0\xi y_1 z_1$, $0\xi y_2 z_2$, $0\xi y_3 z_3$ coincide.

We shall define the angular position of the gyroscope with respect to the base and the angular position of the base with respect to the coordinate system $0\xi\eta\zeta$ by A. N. Krylov angles $\alpha$, $\beta$, $\gamma$ and $\psi$, $\theta$, $\varphi$ respectively (Fig. 2).

§2. Equations of Motion of the System. Using Lagrange equations of the second kind we set up the equations of motion of the system whose kinetic energy is written in the form

$$K = \frac{1}{2} \left( |\dot{V}_0|^2 (m_1 + m_2 + m_3) + A_1 (\alpha + \omega_2 \cos \alpha + \omega_3 \sin \alpha)^2 + B_1 (\omega_2 \cos \alpha + \omega_3 \sin \alpha)^2 + C_1 (-\omega_2 \sin \alpha + \omega_3 \cos \alpha)^2 + (A_2 + A_3) (\omega_2 \cos \alpha + \omega_3 \sin \alpha) \cos \beta + (\omega_2 \sin \alpha \cos \alpha - \omega_3 \sin \alpha \sin \gamma) \sin \beta + (\omega_2 \cos \alpha - \omega_3 \sin \alpha \cos \beta)^2 + C_2 (\omega_2 + \omega_3 \sin \alpha \cos \beta) \sin \beta + (\omega_2 \cos \alpha - \omega_3 \sin \alpha \cos \beta)^2 + C_3 (\omega_2 + \omega_3 \sin \alpha \cos \beta) \sin \beta + (\omega_2 \cos \alpha - \omega_3 \sin \alpha \cos \beta)^2 + \gamma \Delta (V_1 \cos \beta (\omega_2 \cos \alpha + \omega_3 \sin \alpha + \dot{\beta}) + V_2 (\dot{\beta} \sin \alpha + \omega_2 \sin \alpha) \cos \beta - (\omega_2 + \omega_3 \sin \alpha \cos \beta)$$
Here $A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3$ are the moments of inertia of the mass of the outer ring, the casing, and the rotor respectively with respect to the systems $0x_1y_1z_1, 0x_2y_2z_2, 0x_3y_3z_3$; $m_1, m_2, m_3$ are the masses of the outer ring, the casing, and the rotor; $\vec{\omega} = [\omega_x, \omega_y, \omega_z]$ is the angular velocity vector of the base and $\vec{V}_0 = [V_x, V_y, V_z]$ is the linear velocity vector of the point $0$ in the system $0xyz$; $\vec{r} = [0, 0, -\Delta]$ is the radius vector of the shift of the center of gravity of the rotor in relation to the fixed point of the gyroscope in system $0x_3y_3z_3$.

The potential energy of the system has the form

$$p = -m_3g\Delta(-\sin \psi \cos \theta \sin \beta - \sin \theta \sin \alpha \cos \beta + \cos \theta \cos \phi \cos \alpha \cos \beta),$$

where $g$ is acceleration due to earth's gravity.

Applying the well-known Lagrange scheme to expressions (2.1) and (2.2) we obtain the differential equations of motion of the system:

$$-V_z([\dot{\beta} \cos \alpha + \omega_z) \sin \beta + (\omega_x + \dot{\alpha}) \sin \alpha \cos \beta]).$$

(2.1)

where $\Delta$ is the radius of the gyroscope system $0x_3y_3z_3$.