ON ONE CASE OF BUCKLING OF A FLEXIBLE SHAFT

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In automatic and remote control systems as well as others containing flexible shafts as transmitters, buckling of the shaft occurs when a specific torque is exceeded. This phenomenon degrades the qualitative indices of the system or generally impedes its normal operation. Up to now the case of buckling of a flexible shaft when its axis is a straight line has been investigated [4, 5].

The case of buckling of a flexible shaft whose axis is the arc of a circle is examined in this paper.

We solve the problem under the following assumptions, which reflect the actual conditions of flexible shaft operation schematically.

1. The shaft is placed within a "stiff" shell, whose axis is the arc of a circle radius \( R \) (Fig. 1). We assume that the shaft axis was also the arc of a circle prior to emplacement in the shell, but of another radius \( R_0 \); in particular, the shaft could initially be rectilinear. Such an assumption can be made since the flexible stiffness of the shell (the casing) is incomparably greater than the flexible stiffness of the shaft.

2. The sections, plane and normal to the axial line prior to deformation, remain the same even after deformation. Because of the strain due to bending and torsion, a definite distortion of the shaft cross-section will occur which we shall not take into account since it is negligible for large \( R \).

3. We consider the shaft deformations to be elastic. Under these assumptions, the theory of a curved rod of slight curvature can be applied to the shaft. We shall determine the location of the section along the length of the rod by the central angle \( \varphi_L \).

Let us take an Oxyz coordinate system, whose origin is placed at the center of gravity of the section, at an arbitrary cross-section of the rod, and let the z axis be directed along the tangent to the rod axis.

Considering the deformations and displacements originating under the effect of a torque \( M_0 \), we obtain the following expressions for the components of the moments at an arbitrary section of the shaft [3]:

\[
M_x = M_0 \frac{\sin a \varphi_L}{\sinh a \varphi_L}; \quad M_y = M_0 \frac{\sinh a (\varphi_L - \varphi)}{\cosh a \varphi_L};
\]

\[
M_z \approx B \left( \frac{1}{R} - \frac{1}{R_0} \right).
\]

Here \( a = \sqrt{BR/CR_0} \); \( B, C \) are, respectively, the bending and torsion stiffness of the shaft, \( R_0, R \) are the radii of initial curvature before and after stowing the shaft into the shell, and \( \varphi \) is the angle of rotation of the shaft cross-section around the tangent to the axis.

Fig. 1

Fig. 2


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Now, let us investigate the buckling phenomenon for a curved rod by using a static method [2]. To this end, let us trace the displacement of an arbitrary point \( A \) in the rod cross-section whose polar coordinates are \( \rho \) and \( \alpha \) (Fig. 2).

Let us draw a new moving system \( O_{1}x_{1}y_{1} \). As the section rotates through the angle \( \psi \) this point takes on a new position determined by the coordinates \( \rho \) and \( \psi + \alpha \), where the angle \( \psi + \alpha \) will have a different value for different meridian sections \( \varphi \).

After the rod has buckled, the shape of its axis will go from a plane curve into a curve of double curvature. Let us represent its curvature as the vector \( \vec{k} \) along the binormal (Fig. 2). The initial curvature \( 1/R \) of the rod axis is a vector coincident in direction with the negative direction of the \( y \) axis.

The change in curvature at a point \( A \) with the coordinate \( \varphi \) will be

\[
\Delta k_{x1} = k_{x1} + \frac{\sin \alpha}{R}; \quad \Delta k_{y1} = k_{y1} + \frac{\cos \alpha}{R},
\]

where \( k_{x1}, k_{y1} \) are the projections of the curvature of a three-dimensional curve on the \( y_{1}z \) and \( x_{1}z \) planes.

Multiplying the change in curvature by the appropriate bending stiffness, we obtain the bending moments. Then equating them to the corresponding \( M_{x} \) and \( M_{y} \) components, we have

\[
-M_y \sin (\psi + \alpha) - M_x \cos (\psi + \alpha) = B \left( k_{x1} + \frac{\sin \alpha}{R} \right);
-M_y \cos (\psi + \alpha) + M_x \sin (\psi + \alpha) = B \left( k_{y1} + \frac{\cos \alpha}{R} \right),
\]

which can be rewritten as

\[
B \left( k_{1} - \frac{\cos \psi}{R} \right) = M_{y}; \quad B \left( k_{2} + \frac{\sin \psi}{R} \right) = M_{x},
\]

where \( k_{1}, k_{2} \) are projections of the curvature of the shaft axis on the plane of the undeformed axis, and on the surface of a provisional cylinder which always adjoins the \( z \) axis, respectively.

Let us find that critical moment which causes buckling. Let us assume that the deviation of the shaft axis from the initial circular shape after deformation is small. The equations of the strain state can hence be written thus [1]:

\[
\frac{d^2x}{dq^2} + x = R^2 \left( \frac{1}{R} - k_{1} \right); \quad \frac{d^2y}{dq^2} = R^2 k_{2}
\]

\((x, y)\) are the elongation and deflection of the radius \( R \).

The solutions of (5) are

\[
x = C_{1} \cos \varphi + C_{2} \sin \varphi + R^2 \int_{\varphi}^{\varphi} \sin (\varphi - \varphi) \left[ \frac{1}{R} - k_{1}(\varphi) \right] d\varphi;
\]

\[
y = C_{3} + C_{4} \varphi + R^2 \int_{\varphi}^{\varphi} \int_{\varphi}^{\varphi} \frac{k_{2}(\varphi)}{dq} dq d\varphi
\]