of the liquid: its surface-tension coefficient and relaxation and retardation times. The increase in surface
tension leads to a decrease in the value \( \alpha_0 \), and increasing \( \Delta M \) produces an increase in this value.

It is known [5] that if polymer molecules are represented by the dumbbell model, the defining equations
of a dilute solution of polymer molecules may, with the appropriate interpretation of the constants, be repre-
sented in the form of the Oldroyd equations (1.2). According to estimates cited in [7], the values of the relaxa-
tion number \( M_1 \) in the case of the flow of a polymer solution are of the order of \( 10^{-2} \). Since \( M_1 > M_2 \), calcula-
tions with the aid of the formula (3.8) may be limited to the case when \( \Delta M \) is also of the order of \( 10^{-2} \).

For instance, if \( \beta = 0.25, \Delta M = 0.01, \) and \( \beta' = 0.1, \) then the critical wave numbers for the flow on the outer and inner cylinder surfaces are equal, respectively, to 0.39 and 0.44. In the case of the flow of a New-
tonian liquid film of the thickness \( \beta = 0.25 \) these values will be lower: 0.20 and 0.33. Thus, in the case of small values of the wave number the presence of non-Newtonian properties of the medium exerts a destabiliz-
ing influence on the flow.

It should be noted in conclusion that, as in the case of Newtonian liquids [1], increasing the angular rota-
tion velocity of the cylinder \( \Omega \) stabilizes the flow on the inner surface and leads to destabilization of the flow
on the outer cylinder surface.

LITERATURE CITED

6, 96-100 (1975).
4, 544-547 (1967).

STABILITY IN FILM FLOW OF A VISCOUS FLUID
IN A CENTRIFUGAL FORCE FIELD

F. M. Gimranov, N. Kh. Zinnatullin, and F. A. Garifullin

Fluid flow in a centrifugal force field is encountered rather extensively in technology – in rotors of cen-
trifugal machines, pulverizers, mixers, granulators, etc. However, the question of the stability of laminar
flow of the fluids has not been studied thus far.

We consider the flow of a viscous, incompressible, homogeneous fluid flowing in film form over the sur-
face of a rotating flat disk. The following assumptions were made in solving the problem:

1) the angular rate of rotation of the fluid was equal to the angular velocity \( \omega \) of the disk;

2) flow was symmetric with respect to the axis of rotation;
3) the thickness of the fluid film was considerably less than the corresponding radius of rotation;
4) the coordinate system was rigidly fixed in the rotating disk;
5) the abscissa coincided with the free surface of the flowing fluid;
6) the curvature of the free surface was insignificant.

Using the assumptions made, an expression was obtained for the determination of the longitudinal component of the velocity
\[ \bar{U}_x = \frac{\partial^2 x}{\partial \bar{y}^2} (\delta_0^2 - \delta^2). \]  

We investigate the stability of film flow for a viscous fluid in a centrifugal force field with respect to small two-dimensional perturbations assuming the validity of the Squire theorem [3].

Let
\[ \begin{align*}
\bar{v}_x &= \bar{U}_x + \bar{v}_x (\tau, x, y); \\
\bar{v}_y &= \bar{v}_y (\tau, x, y); \\
p &= \bar{P} + \bar{p} (\tau, x, y),
\end{align*} \]

where \( \tau \) is time; \( \bar{U}_x \) and \( \bar{P} \) are the velocity and pressure of the main flow, and the primes denote the components of the velocity and pressure of the perturbing motion.

The flow studied is axisymmetric. However, as shown by numerous experiments, the angle \( \beta \) of the slope of the free surface with respect to the surface of the disk is only \( 5^\circ - 10^\circ \) at most [2]. In a small region of variation of radius from \( x \) to \( x + dx \), therefore, the flow can be assumed plane-parallel and it can be discussed in a Cartesian coordinate system [1]. Under this assumption, one can neglect the normal component \( V \) of the velocity in the main flow and the dependence of the longitudinal \( U \) on \( x \).

It was shown [4, 5] that keeping the quantity \( V \) and the dependence of \( \bar{U} \) on \( x \) in the calculations does not have a significant effect on the study of stability.

Then the linearized differential equations describing flow along the surface of the disk take the form
\[ \frac{\partial^2 \bar{v}_x}{\partial \tau^2} + \bar{U} \frac{\partial \bar{v}_x}{\partial x} + \bar{v}_x \frac{\partial \bar{U}}{\partial y} = - \frac{1}{\rho} \frac{\partial \bar{p}'}{\partial x} + \nu \left( \frac{\partial^2 \bar{v}_x}{\partial x^2} + \frac{\partial^2 \bar{v}_x}{\partial y^2} \right); \]  
\[ \frac{\partial \bar{v}_y}{\partial \tau} + \bar{U} \frac{\partial \bar{v}_y}{\partial x} = - \frac{1}{\rho} \frac{\partial \bar{p}'}{\partial y} + \nu \left( \frac{\partial^2 \bar{v}_y}{\partial x^2} + \frac{\partial^2 \bar{v}_y}{\partial y^2} \right); \]  
\[ \frac{\partial \bar{v}_y}{\partial x} + \frac{\partial \bar{v}_y}{\partial y} = 0. \]

Eliminating \( \bar{p}' \) from Eqs. (3) and (4), we transform to dimensionless variables, taking as characteristic quantities the thickness \( \delta_0 \) of the layer of fluid and the circumferential velocity \( \bar{v}_\phi = \omega x \). We then have
\[ (r, z) = \frac{(x, y)}{\delta_0}; \quad (\bar{v}_x, \bar{v}_y) = \left( \frac{\bar{v}_x}{v_\phi}, \frac{\bar{v}_y}{v_\phi} \right); \]
\[ \rho' = \frac{\bar{p}'}{\rho v_\phi^2}; \quad t = \frac{\tau v_\phi}{\delta_0}; \quad U = \frac{\bar{U}_x}{v_\phi} = b (1 - z^2), \]

where \( b = \omega \delta_0^2 / 2 \nu \).

After appropriate transformations, Eqs. (3), (4), and (5) are written in the form
\[ \frac{\partial^2 \bar{v}_x}{\partial \tau^2} + U \frac{\partial^2 \bar{v}_x}{\partial x^2} + \frac{\partial \bar{v}_x}{\partial x} + \bar{v}_x \frac{\partial \bar{U}}{\partial z} = - \frac{1}{\rho} \frac{\partial \bar{p}'}{\partial x} - \frac{\partial^2 \bar{v}_x}{\partial \tau \partial t} - \frac{\partial^2 \bar{v}_x}{\partial \tau \partial y} - \frac{\partial^2 \bar{v}_x}{\partial y^2}; \]  
\[ -U \frac{\partial^2 \bar{v}_y}{\partial x^2} - U \frac{2}{r} \frac{\partial \bar{v}_y}{\partial r} = \frac{1}{Re} \left( \frac{\partial^2 \bar{v}_y}{\partial r^2} + \frac{2}{r} \frac{\partial \bar{v}_y}{\partial r} + \frac{\partial \bar{v}_y}{\partial \tau} \right); \]  
\[ \frac{\partial \bar{v}_y}{\partial z} + \frac{\partial \bar{v}_y}{\partial y} = 0. \]