CRITICAL LOADS FOR NON-MILDLY-SLOPING ELLIPSOIDAL SHELLS OF UNEQUAL AXES UNDER PRESSURE

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1. Closed Ellipsoidal Shells under a Uniformly Distributed Pressure $p$

It has been shown in [1, 2] that loss of stability close to a zero-moment stressed equilibrium state of a thin linearly elastic isotropic highly convex shell with the formation of dents which are not adjacent to the edge is possible if the equality

$$\max_{(\sigma)} \frac{-Z + [Z^2 + 4K(T_{11}T_{33} - T_{11}T_{33})]^1/2}{K} = \epsilon$$

(1.1)

is attained. Here $\epsilon = 2E\delta/\sqrt{3(1-\nu^2)}$; the maximum is taken with respect to all the interior points of the medial surface $F$ of the shell under discussion; $K$ is the Gaussian curvature of $F$; $Z$ is the surface load acting on the shell; $T_{ij}$ are the stresses caused in the shell by the action of an external load in a subcritical equilibrium state, which are determined from the linear zero-moment theory; $E$ is Young's modulus; $U$ is the Poisson coefficient; and $\delta$ is the thickness of the shell.

At the same time let us assume for the case in question of a uniformly distributed pressure $Z = -p = \text{const}$ that $p > 0$ corresponds to an external pressure. Equation (1.1) determines the main asymptote of the upper critical load in the case of an exceedingly small shell thickness [2].

Let us refer the medial surface $F$ of the ellipsoidal shell to the curvilinear coordinates $\xi_1 = \theta$, $\xi_2 = \varphi$ so that the Cartesian coordinates $x^i$ of the points are specified in the following parametric form:

$$x^1 = a_1 \sin \theta \cos \varphi; \quad x^2 = a_2 \sin \theta \sin \varphi; \quad x^3 = a_3 \cos \theta$$

(0 $\leq \varphi \leq 2\pi$, 0 $\leq \theta \leq \theta_0$).

The equation $\theta = \theta_0$ defines the edge $\partial F$ of the ellipsoid $F$. When $a_1 = 1$, the ellipsoid $F$ changes into a sphere $F$, the Cartesian coordinates of whose points will be $x^i = x^i/a_1$. Without any loss in generality, let us select the numbering of the semiaxes $a_i$ such that

$$a_1 \geq a_2 \geq a_3$$

(1.3)

In addition let us assume in Sec. 1 that $a_2 = a_3$. The Gaussian curvature of the ellipsoid is found from the formula

$$K = \left( a_1 a_2 a_3 \sum_{k=1}^3 b_k(x^3)^k \right)^{-2},$$

(1.4)

where $b_k = 1/a_k^2$.

The stresses $T_{ij}$ caused in the ellipsoidal shell by the action of the pressure $p$ are determined with the use of the method of affine transformations in the linear zero-moment theory of shells [3], pp. 175-184). The final expressions for the stresses assume the following form:

$$T_{11} = \bar{T}_{11}; \quad T_{12} = \bar{T}_{12}; \quad T_{22} = \frac{1}{\epsilon} \bar{T}_{22},$$

(1.5)

Here
\[ \alpha, \beta = 1,2; \quad T_{11}^1 = -b_3 + (b_2 - b_1) \cos 2\varphi; \quad T_{12}^0 = T_{21}^0 = -b_2 - b_1 \cos \sin 2\varphi; \]
\[ T_{13}^0 = -b_3 - (b_3 + b_1 - 2b_2) \sin^2 \theta - (b_2 - b_1) \cos^2 \theta \cos 2\varphi; \]
\[ t_{11}^1 = -s_{12}^2 = \cos 2k\varphi; \quad t_{12}^1 = -s_{12}^1 = -\sin 2k\varphi; \]
\[ t_3^1 = \frac{(a_1^2 \cos^2 \varphi + a_2^2 \sin^2 \varphi) \cos^2 \theta + a_3^2 \sin^2 \theta}{a_1^2 \sin^2 \varphi + a_2^2 \cos^2 \varphi}; \]

For a closed ellipsoidal shell \( \theta_0 = \tau \). It follows from the requirement that the stresses \( T_{k\ell}^{\alpha\beta} \) be bounded at \( \theta = \pi \) that
\[ \lambda_k = 0 \quad (k = 1, 2, \ldots). \] (1.8)
for a closed shell.

Let us substitute (1.5)-(1.8) into Eq. (1.1). Having found the maximum in (1.1), we arrive at the following results.

The main asymptote of the upper critical load \( p_e \) for a sufficiently thin closed ellipsoidal shell under external pressure is determined by the formula
\[ p_e = \frac{b_1 b_2}{b_3 + b_2 - b_1}. \] (1.9)
The loss of stability is accompanied in this case by the formation of dents in the neighborhood of the points of \( \mathcal{F} \) which lie on the smallest semiaxis \( (x^1 = x^2 = 0) \), since the maximum in (1.1) is attained at these points [1, 2].

Loss of stability is possible in the case of internal pressure \( (\varphi < 0) \) if
\[ b_3 - b_2 - b_1 > 0. \] (1.10)
In this connection the main asymptote of the upper critical load is
\[ |p_e| = \frac{eb_1 b_2}{b_3 + b_2 - b_1} , \quad \text{if} \quad 0 < b_3 - b_2 - b_1 < 2b_2; \quad |p_e| = \frac{8eb_1 b_2}{(b_3 + b_2 - b_1)^2} , \quad \text{if} \quad 2b_2 \leq b_3 - b_2 - b_1. \] (1.11)
The loss of stability is accompanied by the appearance of dents in the first case in the neighborhood of points of the ellipsoid \( \mathcal{F} \) which lie on the intermediate semiaxis \( (x^1 = x^3 = 0) \), and in the second case in the neighborhood of points with the coordinates
\[ x^1 = 0; \quad |x^3| = a_3 \frac{b_3 - 3b_2 - b_1}{5(b_3 - b_2)}. \] (1.12)

2. Free Resting against the Plane of Symmetry

Let us consider an ellipsoidal shell (1.5) with edge \( \theta = \theta_0 = \pi/2 \) under uniform external pressure. Let the edge of the shell \( \partial \mathcal{F} \) be free from the action of the bending moment and stresses which act in the plane of the edge; in addition, displacements which are orthogonal to the plane of the edge are equal to zero. Then [2], Sec. 7) there is possible, along with the possibility of stability loss accompanied by the formation of dents not adjacent to \( \partial \mathcal{F} \), a loss of stability in the case of the loads (1.1) which is associated with large deformations (bulging) of the edge \( \partial \mathcal{F} \) and its neighborhood on \( \mathcal{F} \) in the case of loads determined by the equation ([2], Eq. (7.11)]
\[ \max_{(\partial \mathcal{F})} \left( \frac{2T_{11}}{eb_{22}} \right) = \frac{1}{2}, \] (2.1)
where \( b_{22} \) is the normal curvature of \( \mathcal{F} \) in the direction of \( \partial \mathcal{F} \),
\[ b_{22} = -a_1 a_2 (a_1^2 \sin^2 \varphi + a_2^2 \cos^2 \varphi)^{-3/2}. \] (2.2)