
THERMAL STRESS OF NONMONOLITHIC LAMINATED GLASS-REINFORCED PLASTICS

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We propose to analyze the thermal stresses of laminated glass-reinforced plastic whose continuity has been disrupted (without violating the integrity of the reinforcing fibers) under the action of tensile stresses.

The plastic is multilayered (laminated), orthogonally reinforced, and subjected to uniform heating in a steady temperature field. The geometrical and physicomechanical parameters are identical for all layers reinforced in one direction. The material of the layers is linearly elastic and quasi-homogeneous. We investigate the problem in an orthogonal coordinate system Ox1x2x3, whose axes Ox1 and Ox2 coincide with the reinforcement axes of layers i and j, respectively (i, j = 1, 2).

We solve the problem in an approximate setting. We assume that the stresses σij produced in the layers by the perturbation of their stress state due to the presence of discontinuities are negligible in comparison with the other stress components and, accordingly, we disregard the coupling of the layers in the direction normal to their surfaces.

The study is based on an approach [4, 6] that permits the stress state of the laminated material to be detailed, i.e., the components of the stress tensor at points of each layer to be expanded into individual terms and then the solution of the problem to be simplified by isolation of the most important of those terms. The sum and substance of the approach is as follows.

We represent the stress-strain relations at points of the layer R = i, j and the "interface" conditions at the boundaries between layers in the form

\[ \varepsilon_{ijm} = \varepsilon_{ijm}^0 - \mu_{ijm} - \alpha_{ijm}^0 \Delta T; \quad \gamma_{ijm} = \alpha_{ijm}^0 \varepsilon_{ijm}; \]

\[ \varepsilon_{ijm}^0 = \varepsilon_{ijm}^0 |_{b}; \quad | \varepsilon_{ijm}^0 | = | \tau_{ijm}^0 |_{b}; \]

where m is the index number of the stress level; \( a_{ij}, a_{jj}, a_{ii} \) are the elastic constants; \( a_{ij} \) is the coefficient of linear expansion; and

\[ \varepsilon_{ijm}^0 = \alpha_{ij}^0 \varepsilon_{ijm}^0; \quad \varepsilon_{ijm}^0 = \alpha_{ij}^0 \varepsilon_{ijm}^0; \]

\[ \varepsilon_{ijm}^0 = \int_{0}^{\sigma} e_{ijm} dx_i + C_i \quad (C_i = 0). \]


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The principal strains \( \varepsilon_{ii}^{(R)} \) are evaluated for \( m = 1 \) and \( \sigma_{jj}^{(R)} = 0 \); the second-level strains \( \varepsilon_{jj}^{(R)} \) for \( m = 2 \), \( i \) replaced by \( j \), and the known stresses \( \sigma_{ij}^{(R)} \); and the third level strains for \( m = 3 \) and the known stresses \( \sigma_{jj}^{(R)} \). The higher levels of strain are determined analogously.

The principal stresses \( \sigma_{ii}^{(R)} \) are determined without regard for disparities in the transverse deformation properties of the layers, and in determining the second-level stresses the effect of the principal components \( \sigma_{ij}^{(R)} \) on the strains in the direction of the axis \( O\bar{x}_j \) is taken into account; accordingly, in determining the third-level stresses the effect of the stresses \( \sigma_{jj}^{(R)} \) is taken into account, and so on.

To obtain the stresses \( \sigma_{ii}^{(R)} \) we solve the planar problem (for the plane \( O\bar{x}_z \)). The total stresses in the layers are found by summation of the stresses of different levels (but in the same direction) obtained by the solution of two problems (problems \( J \) and \( G \), in each of which the coefficient of linear expansion for one of the principal directions is set equal to zero: \( J \) \( \alpha_{ii} \neq 0, \alpha_{ij} = 0 \); \( G \) \( \alpha_{ij} = 0, \alpha_{jj} \neq 0 \). Thus,

\[
\sigma_{ii}^{(R)} = \sum_{K=J}^{G} \sum_{m} \sigma_{ii}^{(R)_{km}} \quad \tau_{ij}^{(R)} = \sum_{K=J}^{G} \sum_{m} \tau_{ij}^{(R)_{km}}.
\]

We point out that agreement (within 1% limits) of the results obtained in the analysis of monolithic glass-reinforced plastic on the basis of the approach described above and according to formal computing schemes, i.e., using the first expressions (1) and (2) without the index \( m \) attached to \( \varepsilon_{ii} \) and \( \sigma_{ii} \), along with the corresponding equilibrium equations, is attained already with just the first two stress levels.

Experimental studies of the state of orthogonally glass-reinforced plastics under tension in the direction of reinforcement [5] have confirmed the fact that the continuity of the transverse layers is violated by the predominant growth of arterial cracks running in a direction almost normal to the surfaces of the layers. The coefficient of variation \( \omega_{ii} \) [2] for the distances \( l_{ij} \) (along the axis \( O\bar{x}_i \)) between discontinuities (cracks) characterizes the maximum dispersion of random fluctuations of \( l_{ij} \) and decreases with increasing stresses to such an extent as to justify the hypothesis that the quantity \( l_{ij} \) is independent of the coordinates of the points in the direction of tension (for the plastic investigated in [5] the value of \( \omega_{ii} \) decreased to \( \sim 8\% \) for stresses not greater than half the tensile strength).

For large stresses the arterial cracks traverse the entire thickness of the transverse layers. If it is allowed that the distance between the diverging edges of the cracks at the center of a transversely oriented layer of glass-reinforced plastic does not exceed, say, \( 1 \times 10^{-4} \) of the layer thickness (for \( \sigma = 150 \) MN/m\(^2\) [5]), then such cracks will have a slit configuration. Consequently, the field of elastic stresses created in the ruptured laminated composite is conveniently partitioned into two fields, one local (near the tip of the slit), which is characterized by ambiguous physical order and therefore cannot be represented analytically at this time, and the other general, associated with the finite extent of the parts of the layers (between slits); the latter field can be analyzed on the basis of the model of a material with intermittent bonds (ruptured layers). In this article we investigate the general field, assuming the spacings of the crack discontinuities to be quasiregular and considering their edges to be plane and normal to the surfaces of the layers as well as to the corresponding reinforcement axes. We assume that a heat field is generated in the structure (by, for example, an internal pressure void) in its stressed state, so that the edges of the slits can be regarded as separated and their surfaces stress-free (the influence of force, as opposed to the given thermal, stress can be taken into account separately by virtue of the postulated linear elasticity of the material).

In the investigation we ignore the temperature dependence of the physicomechanical properties of glass-reinforced plastic.

We analyze the case of violation of the continuity of layers \( j \).

We discuss the solution of problem \( J (\alpha_{ii} = 0, \alpha_{ij} = 0) \).

To determine the principal stresses we write out the system of equations for the planar problem:

\[
\sigma_{ii}^{(R)_{i34}} + \tau_{ij}^{(R)_{i34}} = 0; \quad \tau_{ij}^{(R)_{i34}} + \sigma_{ij}^{(R)_{i34}} = 0.
\]

The above-noted condition \( \sigma_{33} = 0 \) imposes a definite constraint on the stresses \( \tau_{ij} \) and, accordingly, on \( \sigma_{ii} \), because the condition \( \partial^2/\partial x_i^2 + \partial^2/\partial y^2 \sigma_{ii} + \sigma_{33} = 0 \) must be satisfied. To circumvent the indicated conflict we use only the first expression (6), thereby equalizing the element segregated from the layer \( R \) only with respect to forces acting on it in the direction of the axis \( O\bar{x}_i \). The error induced by the condition \( \sigma_{33} = 0 \) and the