We see from the example presented that, subject to certain simplifying assumptions (infinitely rigid transverse ties for \( n_{nk} \equiv \sigma_0 \); minimum number of branches \( n = 2 \)), the problem is solved in analytical form. The solution of the problem for an arbitrary number of branches \( n \) allowing for the development of elasto-viscoplastic deformations in the transverse and shear ties requires the use of an electronic computer.

**LITERATURE CITED**


**RHEOLOGICAL EQUATION OF STATE OF DILUTE SUSPENSIONS OF RIGID DUMBBELLS HAVING SPHERES AT THE ENDS**

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A dilute suspension of rigid dumbbells (two rigid spheres connected by a rigid tie) constitutes a three-dimensional hydrodynamic model of suspensions of rigid aspherical particles or solutions of polymers, the macroscopic molecules of which possess internal ties not permitting the motion of one part of the molecule relative to another.

The generalization of the structural approach proposed in [7-9] in order to obtain the viscosity of dilute suspensions of rigid spherical particles made it possible [11-13, 15, 16] to find relationships for the viscosity of dilute suspensions of rigid dumbbells having spheres at the ends in simple shear flow, with due allowance for the rotational Brownian motion of the suspended particles. Thus, the authors of [12, 13] established the dependence of the viscosity of such media on the shear velocity and the intensity of the rotational Brownian motion of the suspended particles for the case in which the dumbbell particles moved in the plane of the stream. The viscosity of these media was studied in [15, 16] for a zero shear velocity. A review of articles devoted to this problem may be found in [6].

In this article we shall derive the general rheological equations of state of dilute suspensions of rigid dumbbells with spheres at the ends, without allowing for hydrodynamic interaction between the ends of the suspended particles. As in the case of deriving the rheological equations of state of dilute suspensions of rigid ellipsoids of revolution [3-5], we shall use the structural-continuum approach.

1. **Rate of Energy Dissipation in Unit Volume of Suspension**

Let the dimensions of the suspended dumbbell particles, regarded as possessing a zero buoyancy, be such that the velocity of the solvent within the limits of the particle is a homogeneous function of the coordinates. By the first Helmholtz theorem [1] the velocity \( v_i \) of the liquid at the point A representing the point center of hydrodynamic interaction of the dumbbell with the solvent will then be
where $L$ is the distance between the centers of hydrodynamic interaction of the dumbbell with the solvent; $\omega_{ik}$ is the velocity vortex tensor; $d_{ik}$ is the deformation velocity tensor; $n_k$ is a unit vector characterizing the orientation of the particle relative to the laboratory coordinate system, the origin of which coincides with the center of the axis of the dumbbell $L$ (Fig. 1).

Resolving $v_i$ into two components, in the direction of the particle axis

$$v_i^* = \frac{L}{2} d_{km} n_m n_i,$$

and perpendicular to the axis

$$v_i' = \frac{L}{2} (\omega_{ik} n_k + d_{ik} n_k - d_{km} n_m n_i),$$

we find that the frictional force due to the passage of solvent around the point center $A$ of the particle at a velocity (1.2) is compensated by the action of the force applied to the end $A_1$ symmetrical with respect to the origin of coordinates. The frictional forces due to the flow around the ends of the particle in a direction perpendicular to the axis create a torque

$$\mathbf{M} = L [\mathbf{n} \times \mathbf{F}],$$

where $F_i = \xi (v_i' - 1/2 L n_i)$ is the frictional force acting on one end of the particle (the dot over $n_i$ signifies a total derivative with respect to time); $\xi$ is the translational friction coefficient of the end of the particle interacting with the surrounding liquid as a sphere of radius $a$.

The rate of energy dissipation in unit volume of the suspension under consideration is then given by

$$E = E_p + N_0 \frac{\partial}{\partial t} \left[ \langle v_i'^2 \rangle + \frac{1}{2} \langle v_i' - \frac{L}{2} n_i \rangle^2 \right].$$

Here $E_p = 2\mu d_{ij} d_{ij}$ is the rate of energy dissipation in unit volume of the solvent; $N_0 = c N_A / M$ is the number of suspended particles in unit volume of the suspension; $c$ is the concentration of the suspension (solution); $N_A$ is Avogadro's number; $M$ is the molecular weight of the particle; the brackets $\langle \rangle$ signify averaging carried out with the aid of the angular distribution function of the particle axes, which is a solution of the following equation [2]:

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_i} (F n_i) = 0.$$

Taking account of (1.2) and (1.3) we obtain the dissipation rate in the form

$$E = 2\mu d_{ij} d_{ij} + \frac{1}{2} N_0 L^2 (N_i N_i - 2d_{ij} \langle N_i n_j \rangle + d_{ij} \omega_{ik} \langle n_i n_k \rangle),$$

where $N_1 = n_1 - \omega_{ik} n_k$.

2. Structural-Phenomenological Theory of the Stressed State

According to (1.7) the stress tensor in the medium under consideration should be determined by the equations