Viscoelastic materials are being increasingly used as structural materials [3-8]. Moreover, models of the viscoelastic behavior quite satisfactorily describe the dissipation of energy in numerous traditional materials [4, 7-10]. Structural elements made of such materials are often subjected to intense vibrational loads, as a result of which it becomes necessary to solve problems of forced vibrations of viscoelastic bodies. A problem of this kind is posed by calculations of metal-rubber shock-absorbers whose design is based on the analysis of the transmission coefficients, impedances, etc. [8, 9]. Calculations of the stress-strain state are also of a great importance. Apart from the intrinsic importance of such calculations, they constitute the first stage of the determination of the temperature field produced by spontaneous heating as a result of internal friction.

A review of investigations concerned with the calculation of transmission coefficients and impedances in bars and plates is given in [8, 9]. The character of the difficulties presented by such problems is clearly illustrated in [9] where a circular plate of a constant thickness periodically loaded by a concentrated force is considered. The analytical difficulties become, in fact, insurmountable when the geometry of bars, plates, or shells becomes more complicated, or when various heterogeneities are taken into account. In this article a wide range of problems of forced vibrations of viscoelastic elements (which are reduced to solving ordinary differential equations) is investigated with the aid of Godunov's method of discrete orthogonalization [1] which ensures consistency of the calculating process. The calculation algorithm and a typical program for realizing this method on a type M-220 electronic digital computer were worked out in [2]. Such an approach makes it possible to carry out calculations of transmission coefficients, impedances, stress-strain states, and temperature fields (due to spontaneous heating) of complex viscoelastic systems (typified by bars, plates, and shells, i.e., two- and three-dimensional bodies) taking into account various heterogeneities and anisotropic properties. The fact that inertia forces are taken into account considerably increases the frequency interval considered.

Neglecting the starting conditions (whose influence in the case of viscoelastic elements subjected to periodic loads is significant only during a short period of time), introducing combined mechanical characteristics, and separating the variables with respect to parts of the coordinates, let us reduce the problem of the stress-strain state of viscoelastic bodies to a linear boundary problem relative to the real and virtual parts of the amplitudes of the resolvent functions

\[ \frac{dy}{dz} = A(z) y(z) + f(z) \quad (z_0 < z < z_n) \]

with the boundary conditions

\[ B_1 y = C_1 \quad \text{for} \quad z = z_0; \]
\[ B_2 y = C_2 \quad \text{for} \quad z = z_N, \]

where \( y = (y_1, y_2, \ldots, y_n) \) is the unknown vector function; \( A = \|a_{ij}(z)\| \) is a specified square matrix of order \( n \); \( B_1 = \|b_{1j}\| \), \( B_2 = \|b_{2j}\| \) denote specified rectangular matrices of orders \( k \times n \) and \( (n-k) \times n \) (\( k < n \)), respectively; \( f(z), C_1, \) and \( C_2 \) are specified vectors; \( k \) is the number of the left boundary conditions.

As a result of the introduction of complex moduli and complex resolvent functions the order of the set (1) and boundary conditions (2) and (3) will be twice that obtaining in the case of an elastic material.
Let us consider some examples.

Example 1. Let us solve the problem of forced longitudinal vibrations of a viscoelastic bar at one end \(x = 0\) of which a known periodic force of the amplitude \(F_0\) is acting and at the other end \(x = l\) we have a concentrated mass \(M\). For this problem we have

\[
\begin{align*}
    b_1 &= \frac{du_1}{dx} ; \\
    b_2 &= \frac{du_2}{dx} ; \\
    b_3 &= u_1 ; \\
    b_4 &= u_2 ; \\
\end{align*}
\]

\[
\begin{align*}
    a_{13} &= a_{14} = -a ; \\
    a_{23} &= a_{24} = b ; \\
    a_{41} &= 0 \quad \text{(i, j = 1, 2; i, j = 3, 4)} ; \\
    b_{11} &= b_{12} = 1 ; \\
    b_{21} &= b_{22} = 8 ; \\
    b_{31} &= b_{32} = -\gamma a ; \\
    b_{41} &= b_{42} = \gamma b ; \\
    b_{12} &= b_{21} = 0 ; \\
    z_0 &= 0 ; \\
    z_N &= 1 ; \\
    z &= \frac{x}{l} ; \\
    a &= \frac{(ml)^2 E_v/E_o}{1 + \delta^2} ; \\
    b &= a\gamma ; \\
    m &= \left(\frac{\omega_0}{E_o}\right)^{1/2} .
\end{align*}
\]

Here \(u_1\) and \(u_2\) are the real and the virtual parts of the function \(u^* = (AE_0/F_0)u\); \(\omega\) is the angular frequency; \(\rho\) is the material density; \(\delta\) is the damping coefficient; \(M_R\), \(A\), and \(l\) denote, respectively, the mass, cross-section area, and length of the bar; \(E_o\) is the value of the dynamic elasticity modulus \(E_{\omega}\) at a certain fixed frequency \(\omega_0\); \(u\) is the longitudinal displacement.

The force transmission coefficients \(T_m\) and impedances at the excitation point \(I_0\) are given by the formulas

\[
T_m = \frac{V u_1^2(1) + u_2^2(1)}{V u_1^2(0) + u_2^2(0)} ; \\
I_0 = \frac{m^2 l^2}{V u_1^2(0) + u_2^2(0)} .
\]

Figure 1 (continuous curves) and Fig. 2 show the transmission coefficient and the point impedance plotted against the frequency parameter \(ml\) for \(\gamma = 10\), \(E_o = E_{\omega}\), and various damping coefficients \(\delta\).

Example 2. Let us consider the problem of forced transverse vibrations of a hinged bar of a length \(2l\), in the middle of which \(x = 0\) a concentrated force \(F_0 = F_0 \cos \omega t\) is acting. For this problem we have

\[
\begin{align*}
    y_1 &= \frac{d^2w_1}{dx^2} ; \\
    y_2 &= \frac{d^2w_2}{dx^2} ; \\
    y_3 &= \frac{dw_1}{dx} ; \\
    y_4 &= \frac{dw_2}{dx} ; \\
    y_5 &= \frac{d^2w_1}{dx^2} ; \\
    y_6 &= \frac{d^2w_2}{dx^2} ; \\
\end{align*}
\]

\[
\begin{align*}
    y_1 &= \frac{d^2w_1}{dx^2} ; \\
    y_2 &= \frac{d^2w_2}{dx^2} ; \\
    y_3 &= \frac{dw_1}{dx} ; \\
    y_4 &= \frac{dw_2}{dx} ; \\
\end{align*}
\]

The nonzero matrix coefficients assume, respectively, the form

\[
\begin{align*}
    a_{11} &= a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = a_{17} = a_{18} = a_{19} = a_{10} = 1 ; \\
    a_{32} &= a_{31} = a_{41} = a_{51} = a_{61} = a_{71} = a_{81} = 1 ; \\
\end{align*}
\]