APPROXIMATE CALCULATION OF DRUMS OF LIFTING AND TRANSPORT MACHINES WITH PLATE END FACES

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Often the friction pulleys of multirope mine lifting machines and the drums of belt conveyors represent closed circular cylindrical shells with end faces in the form of annular disks rigidly fixed to hubs. The basic load on the shell is an asymmetric radial pressure of the ropes on the friction pulleys or the transport belts on the conveyor drums.

To simplify the calculation, load on the shell is assumed to be uniformly distributed over the semicircle and constant along the generator (Figs. 1a and 2a). Such a load in the circumferential direction is expanded in the Fourier series

\[ q(\theta) = 0.5q + \sum_{n=1,3,\ldots}^{\infty} \frac{2q}{n\pi} \cos n\theta. \]  

To set up the joining condition of the shell with the support disk, we form a basic system by introduction of a hinged connection of the drum elements, while in the place of the omitted constraints a support moment is applied. Taking into account the symmetry of the external load about the vertical, the reaction moment \( M_0 \) is sought in the form of an even function

\[ M_0 = \sum_{n=0}^{\infty} m_n \cos n\theta. \]

To calculate the angles of rotation in the hinged shell we use an approximate solution in terms of the initial parameters [2]. The initial parameters due to the radial load \( q \) are computed from the condition that at the support the displacement \( w_0 \) in the radial direction, the displacement \( v_0 \) in the circumferential direction, the bending moment \( M_0 \) and the longitudinal force \( N_0 \) are zero, while at the midpoint of the shell the angles of rotation \( \phi \), the displacements \( u \) in the longitudinal direction, the transverse force \( Q_0 \), and the shear force \( S \) are zero. From these conditions we find the initial parameters

\[ Q_0 = \frac{qR}{2p} k_1; \quad \phi_0 = -\frac{qR^2}{E_h} k_2; \quad u_0 = \frac{qR^3}{E_h a} k_3; \quad S_0 = \frac{qRn}{2a} k_4, \]

where \( Q_0 \) is the transverse force, \( \phi_0 \) is the angle of rotation, \( u_0 \) is the displacement along the generator, and \( S_0 \) is the shear force.

The coefficients \( k_1 \) and \( k_2 \) of the initial parameters are calculated in terms of the functions of A. N. Krylov, \( \Phi_1 \) of the arguments \( \beta \xi = \frac{1}{3} \sqrt{(R/6)(l/2R)} \) according to the expressions

\[ k_1 = 2 \frac{\Phi_1 \Phi_2 + 4\Phi_4 \Phi_4}{\Phi_1^2 + 4\Phi_4^2}; \quad k_2 = 4 \frac{\Phi_1 \Phi_2 - \Phi_4 \Phi_4}{\Phi_1^2 + 4\Phi_4^2}. \]

From the same expressions we calculate also the coefficients \( k'_1 \) and \( k'_2 \), but in terms of the function \( \Psi_1 \) of the arguments \( \alpha \xi = (n/\sqrt{n^2 - 1})/2\beta \).

In the case of a skew-asymmetric load, when \( n = 1 \) and \( \alpha = 0 \), with the limit values of the functions \( \Psi_1 = 1, \ \Psi_2 = \alpha \xi, \ \Psi_3 = \alpha^2 \xi^2/2, \ \Psi_4 = \alpha^3 \xi^3/6 \) taken into account, we find
Thus, we obtain the angle of rotation on the support in the shell with hinged fixity due to the radial load $q$

$$\varphi_0 = -\frac{a_0 + q_1 \cos \theta}{E\delta} - \frac{q_1^2 \cos \theta}{24ER^2} \sum_{n=3,5, \ldots} A_n \left( k_{1n} + \frac{n^2 k_2}{4\delta R^2} \right) \cos n\theta. \quad (2)$$

For the moment load $m_0 \cos n\theta$ the initial parameters are computed from the condition that at the origin of the coordinates $w_0 = v_0 = N_0 = 0$, $M_0 = -m_0$ for $\xi = 0$, while $\varphi = u = Q_1 = S = 0$ for $\xi = L/2R$.

Having calculated the initial parameters, we obtain the expression of the angles of rotation in the shell at the support due to the support moment

$$\varphi_0 = \frac{m_0 + m_1 \cos \theta}{ER\delta} - \frac{2q_1^2 \cos \theta}{24ER^3} \sum_{n=3,5, \ldots} m_n \left( k_{1n} + k_{1n}^2 + \frac{n^2}{28\delta R^2} \right) \cos n\theta. \quad (3)$$

For the calculation of the annular plate loaded by the contour moment $m_0 \cos n\theta$ we use the solution of the biharmonic equation $\nabla^2 \nabla^2 w = 0$ in terms of the function of the complex variable $z = \rho e^{i\theta}$ [1]

$$w = 2 \text{Re} \{z \phi(z) + \chi(z)\}. \quad (4)$$

Here $\phi(z)$ and $\chi(z) = \int \psi(z)dz$ are analytic resolvent functions which for the doubly connected region are represented in terms of Laurent series

$$\phi(z) = \sum_{n=-\infty}^{\infty} a_n z^n + a_1 \ln z + b_1 \ln z;$$

$$\chi(z) = \frac{dz(z)}{dz} = \sum_{n=-\infty}^{\infty} a_n z^n + b_1 \ln z. \quad (5)$$

Substituting the functions (5) into the deflection expression (4), we find [1]

$$w = 2 [a_1 \rho^2 + (a_1 \rho^3 + a_1 \rho) \ln \rho + c + (2b_1 \rho \ln \rho + a_1 \rho) \cos \theta + \sum_{n=2}^{\infty} (a_{n-1} \rho^{n-1} - a_{n-1} \rho^{n-2} + na_{n-1} \rho^{n+1} - na_{n-1} \rho^{n+2}) \cos n\theta \cos n\theta]. \quad (6)$$

Hence, knowing the cylindrical rigidity $D = E\delta^2 \left( 12(1 - \nu^2) \right)$, from the differential relationships

$$M_\rho = -D \left[ \frac{\partial^2 \varphi}{\partial \rho^2} \right] - \phi \left( \frac{1}{\rho^2} \frac{\partial \varphi}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \varphi}{\partial \rho^2} \right)$$

we obtain the equations of the radial moments $M_\rho$ and the angles of rotation $\varphi$

$$\frac{M_\rho}{2D} = 2a_1 (1 + \nu) + a_1 (2(1 + \nu) \ln \rho - (3 + \nu)) - a_{-1} \rho^2 (1 - \nu)$$

$$+ 2 [(1 + \nu) \delta^2 \rho^{-1} + a_1 (3 + \nu) \rho - a_{-2} (1 - \nu) \rho^{-1}] \cos \theta$$