The linear formulation and methods of solving bending problems of plates with stiffener ribs have been studied well enough [4, 5]. The question of the combined geometrically nonlinear deformation of a plate with a rib has hardly been investigated as is known. Here, one variation of the formulation and method of solving a class of problems devoted to this question is presented.

§1. Fundamental Equations of Flexible Plates

Let us examine a flexible plate of arbitrary outline, reinforced by flexible stiffener ribs which have finite tensile, bending, and twisting stiffness, prior to deformation. Assuming the rib sufficiently thin, let us replace it by the equivalent stiffness characteristics of a slender rod located in the middle plane of the plate [5]. We consider the plate material homogeneous, isotropic.

Let us refer the middle plane (surface) prior to and after deformation to the orthogonal curvilinear coordinates $\alpha, \beta$. The quantities referred to the strain state will be denoted by an asterisk. Let us use the notation: $O(e_\alpha, e_\beta, e_\gamma)$ is a local, left-hand, orthonormalized trihedron on the middle plane of the plate prior to deformation (Fig. 1), $O^*(e^*_\alpha, e^*_\beta, e^*_\gamma)$ is an analogous left-hand trihedron on the strained surface of the plane, and $u_\alpha$ is the displacement vector of points of the plate ($u_\alpha = ve_\alpha + \psi e_\beta + w_\gamma$).

Let the surface or edge forces acting on the plate be of that magnitude for which the plate will admit deflections on the order of several thicknesses during bending, and the squares of the angles of rotation and of the strains small in comparison to one (mean bending). In this case the determination of the stress—strain state of the plates reduces to integrating the Karman equations for the deflection and stress function $F$

$$Dv^2q^{\alpha}\alpha + T_{\alpha\beta}v_{\alpha\beta} + 2T_{\alpha\gamma}v_{\alpha\gamma} + T_{\gamma\beta}v_{\gamma\beta} = q(\alpha, \beta);$$

$$\nabla^2v^2F - Eh(v_{\alpha\beta}^2 - v_{\alpha\alpha}v_{\beta\beta}) = 0. (1.2)$$

Here

$$BT_{\alpha\alpha} = \frac{\partial}{\partial \beta}\left(1 + \frac{\partial F}{\partial \beta}\right) + \frac{1}{A^2}\frac{\partial \beta}{\partial \alpha};$$

$$ABT_{\alpha\beta} = \frac{\partial^2 F}{\partial \alpha \partial \beta} + \frac{1}{B}\frac{\partial \beta}{\partial \alpha} + \frac{1}{A}\frac{\partial \beta}{\partial \alpha};$$

$$v_{\alpha\alpha} = -\frac{1}{A}\left(\frac{\partial \psi_\alpha}{\partial \alpha} + \frac{\psi_\alpha}{B}\frac{\partial A}{\partial \alpha}\right) ; v_{\alpha\beta} = -\frac{1}{A}\left(\frac{\partial \psi_\beta}{\partial \beta} - \frac{\psi_\alpha}{B}\frac{\partial A}{\partial \beta}\right).$$

The direction cosines of the angles between the basis directions $O, O^*$ are presented in Table 1, where

$$\psi_n = \frac{1}{A} \frac{\partial \omega}{\partial \alpha} \text{ (curl)}; \quad \nabla^2 = \frac{1}{AB} \left[ \frac{\partial}{\partial \alpha} \left( B \frac{\partial}{\partial B} \right) + \frac{\partial}{\partial B} \left( A \frac{\partial}{\partial B} \right) \right].$$

and $A, B$ are Lamé parameters.

§ 2. Fundamental Relationships, Equilibrium Equations of Flexible Curvilinear Rods

The derivation of the fundamental relationships governing the equations of two- or three-dimensional curved rods has been done by G. Kirchhoff, A. Clebsch, B. Saint-Venant [2], E. L. Nikolai [3], and A. I. Lur'e [1]. In order to obtain simplified relationships and for clarity, let us apply a method somewhat different from [2, 3] to derive the governing equations.

Prior to deformation let $n_2$ characterize the position of points of the rod axis by the longitudinal coordinate $s$. Let us construct an orthonormal trihedron $O_1(e_1, e_2, e_3)$ of left-hand orientation at a given point of the axis; we hence direct $e_2$ along the tangent to the axis, and $e_1, e_3$ parallel to the principal axes of inertia of the rod cross-section (Fig. 2). For simplicity in the computations, we consider the section to have two axes of symmetry.

The derivational formulas

$$\frac{d e_1}{ds} = k_1 e_1; \quad \frac{d e_2}{ds} = -k_1 e_2; \quad \frac{d e_3}{ds} = 0, \quad (2.1)$$

are valid for the basis $O_1$, where $k_1$ is the curvature of the rod axis.

Using the notation $u_0 = u_0 e_1 + v_0 e_2 + w_0 e_3$ is the displacement vector of points of the rod axis; $r(s), r^*(s)$ radius-vector of points of the axis prior to and after deformation, we obtain

$$ds^2 - ds^2 = dr^* \cdot dr^* - dr \cdot dr = 2e_2 ds^2. \quad (2.2)$$

Here

$$\epsilon = \epsilon + \frac{1}{2} \epsilon^2 + \frac{1}{2} \psi^2 + \frac{1}{2} \theta^2; \quad \omega = \frac{du_0}{ds} - k_1 v_0; \quad (2.3)$$

Let us also construct a left-hand orthonormal trihedron $O^*_1(e^*_1, e^*_2, e^*_3)$ on the deformed axis according to the rule: We extract three mutually perpendicular material fibers directed along the directions of the basis $O_1$ provisionally prior to deformation. Then we direct the direction of $e^*_2$ along the tangent to the rod axis, and draw the plane $\Sigma^*$ through the direction $e^*_2$ and a fiber element directed along the direction $e^*_3$ prior to deformation. Let us draw the direction $e^*_1$, $e^*_3$ with an arbitrarily selected positive direction in $\Sigma^*$; let us define the direction $e^*_1$ as the product $e^*_1 \times e^*_2$, where we consider $e^*_2$ directed so that the trihedron $O^*_1(e^*_1, e^*_2, e^*_3)$ would have left-hand orientation. The axes thus constructed are called the principal axes of bending and twisting [2]. We consequently have