systems of compromises are those realized using the principles of quasiequality, valid reduction, and minimal deviation from the utopic point. The approach proposed for the solution of optimization problems for rod and thin-walled systems allows the factor of technological feasibility to be taken into account ($F_i = \text{const, } q < m$).

**LITERATURE CITED**


**EFFECT OF THE COMPLIANCE OF FOUNDATION SUPPORTS ON THE MOTION OF A CENTRIFUGAL PUMP IMPELLER**

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This study deals with the motion of a centrifugal pump impeller. We consider the effect of the elasticity and damping characteristics of the foundation supports on the stability of synchronous precession of an out-of-balance impeller with a single-groove seal mounted on an elastic and inert base. For establishing the stability criteria, we apply the method of averaging to the periodic coefficients in the perturbation equations and then quasinormalize those equations.

The dynamic model of such an impeller is a flexible weightless shaft of stiffness $c_1$ with a disk of mass $m_1$ at the center of the span. The impeller is mounted on a perfectly rigid plate of mass $m_2$ resting on inertialess spring supports. All masses and stiffnesses of the system are assumed to be symmetric with respect to the plane of the disk, with the stiffness $c_2$ of the foundation supports the same in the horizontal plane and in the vertical plane.

The displacement of the disk, relative to the center of the seal, during flexure of the shaft produces a circumferential pressure gradient in the fluid around the disk and a hydrodynamic net friction force normal to the plane of disk flexure. This force can, under certain conditions, cause self-excited asynchronous precession of the impeller [2].


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The projections of this hydrodynamic friction force on the axes of \( \xi \eta \) coordinates rotating at an angular velocity \( \dot{\gamma} \) are, in the case of a laminar flow of the fluid \([2]\),

\[
P_{\xi} = 0; \quad P_{\eta} = \frac{\mu R R p e}{8} \left( \frac{\omega}{2} - \dot{\gamma} \right) \left( 1 - \chi^2 \right)^{-\frac{3}{2}},
\]

(1)

where \( \mu \) is the dynamic viscosity of the fluid; \( R, I, \delta \), respectively, the radius, length, and mean width of the 0-seal groove; \( \omega \), impeller speed; \( \dot{\gamma} \), rate of impeller precession; \( e \), displacement of the impeller center from the seal center; and \( \chi = e/\delta \), eccentricity.

The reaction of a layer of viscous fluid is, in the stationary system of coordinates \( x\eta y \),

\[
P = - k f (\chi) \left[ z_1 - \dot{z}_2 - i \frac{\omega}{2} (z_1 - z_2) \right],
\]

(2)

where the complex coordinates \( z_\beta = x_\beta + iy_\beta \) characterize the location of masses \( m_1 \) and \( m_2 \), respectively:

\[
k = \frac{\mu R R p e}{8}; \quad f (\chi) = n (1 - \chi^2)^{-\frac{3}{2}}; \quad \chi = \frac{|z_1 - z_2|}{\delta}.
\]

The motion of the impeller—foundation system can be described by the equations

\[
m_1 \dddot{z}_1 + c_1 (z_1 - z_2) + k f (\chi) \left[ \dddot{z}_1 - \dddot{z}_2 - i \frac{\omega}{2} (z_1 - z_2) \right] = e m_1 \omega^2 \exp i \omega t;
\]

\[
m_2 \dddot{z}_2 + c_2 (z_1 - z_2) + k f (\chi) \left[ \dddot{z}_1 - \dddot{z}_2 - i \frac{\omega}{2} (z_1 - z_2) \right] + \alpha z_2 = 0,
\]

(3)

which take into account the forces of friction appearing during the deformation of the foundation supports. Here \( \varepsilon \) is the static eccentricity and \( \alpha \) is the damping coefficient of the foundation supports.

When the system is far from resonance, then the forces of external and hydrodynamic friction hardly affect the amplitude and the phase of forced vibrations. It is, therefore, permissible to assume that at impeller speeds sufficiently far from the critical speed the transient forced vibration of the system can be described by particular solutions to Eqs. (3)

\[
z_{10} = A_1 \exp i \omega t; \quad z_{20} = A_2 \exp i \omega t,
\]

(4)

with the amplitudes \( A_1 \) and \( A_2 \) defined as

\[
A_1 = \frac{e m_1 \omega^2 c_1 + c_2 - m_2 \omega^2}{(c_1 - m_2 \omega^2)(c_1 + c_2 - m_2 \omega^2) - c_1^2};
\]

\[
A_2 = \frac{e m_1 \omega^2 c_1}{(c_1 - m_2 \omega^2)(c_1 + c_2 - m_2 \omega^2) - c_1^2}.
\]

(5)

For analyzing the stability of the impeller motion, we let \( z_\beta = z_{10} + w_\beta \) \((s = 1, 2)\) and regard \( w_\beta = u_\beta + iv_\beta \) as small perturbations of the solutions \( z_{10} = A_1 \exp i \omega t, \) \( z_{20} = A_2 \exp i \omega t \). Then expansion of the nonlinear terms in Eqs. (3) into Taylor series yields a system of variational equations with periodic coefficients

\[
m_1 \dddot{w}_1 + c_1 (w_1 - w_2) + k f (\chi_0) \left[ \dddot{w}_1 - \dddot{w}_2 - i \frac{\omega}{2} (w_1 - w_2) \right]
\]

\[
+ \frac{1}{2} k \omega \left[ \chi \frac{\partial f (\chi)}{\partial \chi} \right] \left[ (u_1 - u_2) \cos \omega t + (v_1 - v_2) \sin \omega t \right] \exp i \omega t = 0;
\]

\[
m_2 \dddot{w}_2 + c_2 (w_1 - w_2) + k f (\chi_0) \left[ \dddot{w}_1 - \dddot{w}_2 - i \frac{\omega}{2} (w_1 - w_2) \right]
\]

\[
- \frac{1}{2} k \omega \left[ \chi \frac{\partial f (\chi)}{\partial \chi} \right] \left[ (u_1 - u_2) \cos \omega t + (v_1 - v_2) \sin \omega t \right] \exp i \omega t = 0.
\]

(6)

The brackets with a 0 subscript in Eqs. (6) indicate that the expressions inside refer to \( w_\beta = 0 \) \((s = 1, 2)\).

Equations (6) with the substitution \([1]\)

\[
w_i = \sum_{j=1}^{2} a_{ij} q_j \quad (q_j = r_j + i p_j)
\]

(7)