1 and 2 correspond to shells of types 1 and 3. The solid lines represent the observed values and the dot-dash lines are from numerical analysis.

These results show that the general pattern of the strain in the outer layer of such a shell is much the same for all types of reinforcement.

The theoretical and experimental results are in satisfactory agreement over the general pattern of the state of strain, which indicates that one can calculate the state of stress and strain for a shell of this type by our method with an accuracy adequate for practical purposes, and in particular that it is possible to apply certain correction factors at the design stage, which may be derived empirically, to evaluate the performance of a particular type of reinforcement and to define the best parameters.

**LITERATURE CITED**


**OPTIMUM DESIGN OF ELASTIC SYSTEMS**

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At present the finite-element method is being widely used to investigate the stress-strain state of complex continuous systems [3-5]. Because of its universality, this method has also been applied to optimum design problems [7]. The method is still not widely enough used in optimization problems with stability constraints. This results from the necessity of operating with high-order matrices for each set of variable parameters with limited internal computer storage.

Computational experiments [4, 5] showed that the time to perform one calculation varied from a few minutes to several hours depending on the speed of the computer and the dimensions of the matrix. Therefore one of the urgent problems of optimum design is the construction of algorithms with a minimum number of recursions to finite-element procedures.

In the present paper we use the example or a ribbed plate to present a new approach to the solution of the problem of optimum design by the finite-element method. With this approach, loss of stability can be taken into account in the analysis of complex systems.

1. We consider a rectangular ribbed plate stressed in its plane by compressive boundary forces. To determine the critical loads, the structure is represented as a discrete model of a net of rectangular elements with ribs around the edge. The overall stiffness and stability matrices A and B of an n-th order system are formed automatically from the stiffness and stability matrices of the individual elements.

We denote by \(x_1, x_2, x_3\), respectively, the thicknesses of the plate, the longitudinal ribs, and the transverse ribs. Then the problem consists of determining the vector \(x\) with components \(x_1, x_2, x_3\) which minimizes the volume of the structure

\[
V(x) = h_1 x_1 + h_2 x_2^2 + h_3 x_3^2
\]  

subject to strength constraints.
geometrical constraints

\[ a_{ij}(x) \leq a_{ij}, \quad (1.2) \]

and stability constraints

\[ x_{i_{\min}} \leq x_j \leq x_{i_{\max}}, \quad (1.3) \]

\[ P^*(x) \geq P, \quad (1.4) \]

where \( k \) and \( k_i \) are coefficients taking account of the number of longitudinal (transverse) ribs and the ratio of the height of a rib to its thickness; \( \sigma_0(x) \) is the stress from the active load;

\[ P^*(x) = \lambda(x) \left( h + \frac{kr_2}{x_1} \right), \quad (1.5) \]

\( \lambda(x) \) is the minimum eigenvalue of the regular family of quadratic forms \( A(x) - \lambda(x)B(x) \).

We propose to solve problem (1.1)-(1.4) by using an extrapolation algorithm in which, in contrast with [6], at each \( m \)-th step the subsidiary problem of nonlinear programming (1.1)-(1.3), (1.6) is solved.

\[ P_{ij}^*(x, x^m) \geq P \quad (i = 1, q_i < n), \quad (1.6) \]

where \( P_{ij}^*(x, x^m) \) is defined in accordance with (1.5) and \( \lambda(x) \) is replaced by a linear approximation of the corresponding \( l \)-th eigenvalue in the neighborhood of the point \( x^m \).

The possibility of converting to the subsidiary problem in the neighborhood of the point \( x^m \) follows directly from the fact that the functions of the original problem are continuously differentiable. We write the penalty function for problem (1.1)-(1.4) in the form

\[ \Phi(x) = V(x) + NG(x), \quad (1.7) \]

where \( G(x) = [P - P^*(x)]^2 + |a_{ij}(x) - \sigma_{ij}|^2 + \sum_{j=1}^{n} (|x_j - x_{j_{\max}}|^2 + |x_{j_{\min}} - x_j|^2) \), and

\[ N > 0, \| \cdot \|_\infty = \max |\cdot|_\infty. \]

At the point \( x^m \) the gradient \( \nabla \Phi(x^m) \) is the same as the gradient of the penalty function of the subsidiary problem. Since the solution of the problem of nonlinear programming and the minimum of the corresponding penalty function are related in a certain sense [9], it follows from the equality of the gradients that the solutions of the main and subsidiary problems agree to first order accuracy.

Suppose the result of solving the subsidiary problem at the point \( x^m \) is \( x^{m+1} \). We minimize \( \Phi(x) \) by an algorithm of the form

\[ x^{m+1} = x^m + \beta_m (x^m - x^0), \]

where \( \beta_m = 2^{-k_m} \); \( k_m \) is the smallest positive integer for which the condition

\[ \Phi(x^m + \beta_m (x^m - x^0)) - \Phi(x^m) \leq \epsilon \Phi(x^m, x^m - x^0) \quad (0 < \epsilon < 1) \]

is satisfied.

The following statements are valid for the sequence \( \{x^m\} \):

1. \( \lim_{m \to \infty} x^m = x^*, \quad \nabla \Phi(x^*) = 0. \)

2. For sufficiently large \( m \) we have \( \| x^{m+1} - x^* \| \leq \epsilon_1 \| x^m - x^* \| \). (\( \epsilon_1 < 1 \)).

Thus, the algorithm presented for constructing the sequence \( \{x^m\} \) converges to the point of the minimum of the penalty function no more slowly than an infinite decreasing geometrical progression with denominator \( \epsilon_1 \).

2. As components of the method proposed for solving the subsidiary problem (1.1)-(1.4) algorithms were developed for solving the subsidiary problem (1.1)-(1.3), (1.6) (discussed in detail in [2]) and for seeking eigenvalues (described in [1]).

The necessity for developing an algorithm for seeking eigenvalues results from the fact that many of the existing methods for solving generalized eigenvalue problems are based on