Optimum Coupling and Efficiency of Dye Lasers*

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Abstract. The optimum coupling for dye lasers has been theoretically studied, taking into account the correct rate equations. The results obtained are applicable also to flashlamp-pumped dye lasers, provided that the duration of the pumping pulse is long compared to the decay times of the upper singlet band and of the triplet band. The behaviour of the output power is also discussed and an expression for the efficiency of the laser is obtained.

Index Headings: Dye lasers – Optimum coupling

The problem of the optimum output coupling has been solved for the case of three and four-level lasers, both when power is taken out at the laser frequency [1] and for intracavity second-harmonic generation [2]. The rate equations for a dye laser are different from those used for a four-level laser, both for the triplet terms and for the terms taking into account the singlet-singlet absorption; then the problem of optimum output coupling for dye lasers must be reconsidered taking into account the correct equations. The purpose of this paper is to solve the problem of the optimum coupling for a cw dye laser, when power is taken out at the laser frequency, and calculate the optimum efficiency. The results obtained will apply also to flashlamp-pumped dye lasers, provided that the duration of the pumping pulse is long compared with the decay times of the upper singlet band and of the triplet band [3].

The rate equations for a dye laser may be written as follows [4]

\[
\frac{dN}{dt} = W_p N_t - B_L N q - N/\tau + B_u N_u q , \quad (1a)
\]

\[
\frac{dN_T}{dt} = k_{ST} N - N_T/\tau_T , \quad (1b)
\]

\[
\frac{dq}{dt} = V B_L N q - K_i q - K_u q - V B_u N_u q - V B_T N_T q , \quad (1c)
\]

where \( N \) is the population of the upper laser band (upper singlet band) per unit volume, \( W_p \) is proportional to the pump rate, \( N_t \) is the number of active molecules per unit volume, the \( B \)'s are stimulated emission (or absorption) coefficients per photon and per molecule. \( B_L = \alpha_L c/V \) where \( \alpha_L \) is the cross section of the transition, \( c \) is the velocity of light in the medium and \( V \) is the mode volume in the active material. \( \sigma_L \) is the stimulated emission cross section of the laser transition, \( \sigma_u \) is the singlet-singlet absorption cross section and \( \sigma_T \) is the triplet-triplet absorption cross section. All cross sections are considered at the laser frequency, \( q \) is the total number of photons in the cavity at the laser frequency, \( \tau \) is the decay time of the upper laser band, i.e. the fluorescence lifetime [5], \( N_T \) is the triplet band population per unit volume, \( k_{ST} \) is the intersystem crossing rate between the upper laser band and the triplet band [5], \( \tau_T \) is the decay time of the triplet band [5]. \( K_i \) accounts for the internal losses of the laser, except the losses due to singlet-singlet absorption and to triplet-triplet absorption; then it accounts for mirrors absorption, scattering, diffraction, etc. \( K_i = \gamma_i c/d \), where \( \gamma_i \) is the fraction of power lost per pass due to the internal losses mentioned above and \( d \) is the length of the active material. In the arrangements normally used for dye lasers \( \gamma_i \) may range from 1 to 2%. \( K_u \) accounts for the power lost by the cavity through the output coupling. In the case of two equally transmitting mirrors \( K_u = \gamma_u c/d \) where \( \gamma_u \) is the transmission of each mirror.
Equation (1) are written under the assumptions, usually accepted for four-level lasers, that the decay time of the lower laser level is very fast and that the number of active molecules per unit volume in the lower singlet band (lower laser band), is not practically altered by the pumping process and can be considered constant and equal to \( N_t \). Moreover (1b) is written under the assumption that, if \( T_2 \) and \( T_1 \) are the upper and lower triplet bands respectively, the \( T_2 \rightarrow T_1 \) relaxation time is so fast that the two bands are always in equilibrium.

The steady state condition is obtained by setting in (1a)–(1c) the time derivatives to zero. Hence

\[
\begin{align*}
W_p N_t - B_L N_t q - N_t/\tau + B_a N_t q &= 0, \quad (2a) \\
K_S N - N_t/\tau_T &= 0, \quad (2b) \\
V B_L N_t q - K_i q - K_a q - V B_a N_t q - V B_T N_t q &= 0. \quad (2c)
\end{align*}
\]

From (2) the following expression for the number of photons in the cavity is obtained

\[
q = \frac{V}{\tau} \left( \frac{1}{(K_i + K_a + B_T k_{ST} \tau_T) / (B_L - B_T k_{ST} \tau_T)} - \left( W_p N_t V B_L \tau - K_i - K_a - V B_a N_t \right) \right).
\]

The number of photons leaving the cavity per unit time is \( Q_u = K_u q \), then:

\[
Q_u = \frac{1}{B_L \tau} \frac{K_u}{K_i + K_a + a(K_i + K_a + V B_a N_t)} \cdot \left( W_p N_t V B_L \tau - K_i - K_a - V B_a N_t \right),
\]

where

\[
B_L = B_{Le} - k_{ST} \tau_T, \quad a = \frac{B_T k_{ST} \tau_T}{B_L - B_T k_{ST} \tau_T} = \frac{\sigma_T k_{ST} \tau_T}{\sigma_L - \sigma_T k_{ST} \tau_T}.
\]

It is worth to notice that

\[
B_L = B_{Le}(1 + a)
\]

and that, if the triplet effects are negligible, \( a \) can be set equal to zero.

It is convenient to put (3) in the following form

\[
Q_u = \frac{1}{B_L \tau} \frac{K_u'}{K_u' + K_i'} \left( W_p N_t V B_L \tau - K_u' - K_i' - V B_a N_t \right),
\]

where

\[
Q_u = Q_u(1 + a) \\
K_u = K_u(1 + a) \\
K_i = \left[ K_i(1 + a) + a V B_a N_t \right].
\]

Now, to find the optimum coupling value for \( K_u \), we must find the maximum of \( Q_u \), and this is obtained by setting

\[
\frac{\partial Q_u}{\partial K_u} = 0.
\]

From (5), after some algebraic manipulations, we obtain the following expression for the optimum coupling value \( K_{u0} \) of \( K_u \)

\[
K_{u0} = \sqrt{\left( W_p N_t V B_L \tau - V B_a N_t \right) K_i - K_i}
\]

and the equivalent expression for the optimum value \( K_{u0} \) of \( K_u \)

\[
K_{u0}(1 + a) = \sqrt{\left( W_p N_t V B_L \tau - V B_a N_t \right) \left( K_i(a + 1) + a V B_a N_t \right) - \left( K_i(1 + a) + a V B_a N_t \right)}.
\]

It is convenient to write (6a) in terms of measurable quantities and in a more convenient dimensionless form; to do this we calculate now the value \( W^* \) of \( W_p \) at threshold when \( K_u \) is zero, i.e. when the laser cavity is made with 100% reflecting mirrors; with the help of (2a)–(2c) we obtain

\[
W^* = \frac{K_i + V B_a N_t}{N_t V B_L \tau}.
\]

We define now the quantity \( x = W_p/W^* \), and obtain for \( x \) the expression

\[
x = \frac{W_p N_t V B_L \tau}{K_i + V B_a N_t},
\]

or the equivalent expression

\[
x = \frac{W_p N_t V B_L \tau}{K_i + V B_a N_t}.
\]

Being \( W_p \) proportional to the pump light intensity, \( x \) is just the ratio between the outputs of a phototube monitoring the pump light in the absorption band of the active material, first at the desired pump intensity, and then at threshold when the actual mirrors are substituted by highly reflecting mirrors. It is then an easily measurable quantity. Let us now define the