Theory of Self-Locking Phenomena in the Pressure Broadened Three-Mode He-Ne Laser

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Abstract. Taking account of pressure broadening the coefficients in Lamb's third-order amplitude and phase determining equations for three modes are evaluated in the Doppler limit, and comparison is made with other theories. After expanding the coefficients to first order in the detuning \( \epsilon \) from the linecenter an expression is presented for the frequency range where mode locking will occur.

Using experimental data of several authors for the 6328 Å Ne-transition numerical values are obtained for the pressure dependent decay rates \( \gamma_a, \gamma_b \), of the upper and lower laser level and the linewidth \( \gamma_{ab} \), viz.: \( \gamma_a = 17.7 + (12.6 \pm 0.6)P_{\text{He}} \), \( \gamma_b = 8.3 + (12.6 \pm 0.6)P_{\text{He}} \), and \( \gamma_{ab} = 13.0 + (91 \pm 6)P_{\text{He}} \) MHz, where \( P_{\text{He}} \) is measured in torr.

Index Headings: Lasers - Self-locking - Pressure broadening

Consider a three-mode laser, oscillating at frequencies \( v_1, v_2, \) and \( v_3 \). Due to dispersion in the medium (pushing and pulling) these oscillating frequencies are not, in general, equal to the eigenfrequencies \( \Omega_1, \Omega_2, \) and \( \Omega_3 \) of the cavity, which are equally spaced such that \( (\Omega_2 - \Omega_1) = (\Omega_3 - \Omega_2) = 2(\Omega_2 - \Omega_1) = 0 \). Lamb's [1] amplitude determining equations look like

\[
\begin{align*}
\dot{E}_1 &= \alpha_1 E_1 - \beta_1 E_1^3 - \theta_{12} E_1 E_2 - \theta_{13} E_1 E_3 - (\eta_{23} \cos \psi + \xi_{23} \sin \psi) E_2^2 E_3 \\
\dot{E}_2 &= \alpha_2 E_2 - \beta_2 E_2^3 - \theta_{21} E_1^2 E_2 - \theta_{23} E_2 E_3 - (\eta_{13} \cos \psi - \xi_{13} \sin \psi) E_1 E_2 E_3 \\
\dot{E}_3 &= \alpha_3 E_3 - \beta_3 E_3^3 - \theta_{32} E_3 E_2^2 - \theta_{31} E_3 E_1^2 - (\eta_{21} \cos \psi - \xi_{21} \sin \psi) E_2 E_3^2 E_1
\end{align*}
\]

The phase determining equations are

\[
\begin{align*}
\dot{\phi}_1 &= \Omega_1 + \sigma_1 + \tau_{12} E_2 + \tau_{13} E_3 - (\eta_{23} \sin \psi - \xi_{23} \cos \psi) E_2^2 E_3 E_1^{-1} \\
\dot{\phi}_2 &= \Omega_2 + \sigma_2 + \tau_{21} E_1 + \tau_{23} E_3 + (\eta_{13} \sin \psi + \xi_{13} \cos \psi) E_1 E_3 \\
\dot{\phi}_3 &= \Omega_3 + \sigma_3 + \tau_{31} E_1^2 + \tau_{32} E_2 + \tau_{33} E_3^2 - (\eta_{21} \sin \psi - \xi_{21} \cos \psi) E_2 E_3^2 E_1^{-1} \\
\dot{\psi} &= (2v_2 - v_1 - v_3) t + (2\varphi_2 - \varphi_1 - \varphi_3) \\
\end{align*}
\]

The corrections by Sayers and Allen [2] in the signs of some coefficients in the original Lamb paper have been included. Furthermore, it should be mentioned that \( \xi_{23} \) and \( \xi_{21} \), as given by Lamb, are incomplete even in the zero pressure limit [3].

As a result of the nonlinearities it is possible to generate quanta e.g. with frequency \( \psi = 2v_2 - v_1 - v_3 \), which is close to zero. These quanta couple the three modes, as depicted in Fig. 1. The cross section of the coupling process, proportional to the total number of quanta present, will be strongly enhanced - due to the wellknown denominator \( 1 - \hbar \psi \), following from second-order perturbation theory - if the virtual level \( i \) lies well within the linewidth of level...
Fig. 1. Coupling scheme for three modes. Modes 1 and 3 are coupled to each other and to mode 2 through the virtual level \( i \), the beatnotes and the low-frequency quanta \( h\psi \).

Fig. 2. Frequency spectrum of a three mode laser with combination tones generated in the nonlinear gainmedium. The tone \( v_3 + v_2 - v_1 \) corresponds to the virtual level \( i \) of Fig. 1.

2. This can be achieved by tuning the central mode towards the middle of the inhomogeneous profile: pushing and pulling effects will decrease such that the three modes may become equally spaced, i.e. \( \psi = (v_2 - v_1) - (v_3 - v_2) \to 0 \).

Another way of looking at the phenomenon may be more appealing to radio and circuit engineers: due to the third-order terms in the amplitude equations “combination tones” are generated near the original frequencies, as shown in Fig. 2. One can think of a combination tone as an “injected, external” signal to which the original mode can be locked. This external locking was already noticed in 1927 by Van der Pol [4] as “automatic synchronization” in discussing a self-sustained triode oscillator, and was even known to Huygens who observed that two clocks hung on the same wall could synchronize [5].

The locking phenomenon, which occurs rather suddenly, is observed in the frequency domain through the absence of low-frequency amplitude noise. This noise arises from competition for gain and is expressed in (1)–(3) as \( \psi \)-oscillations. In the locking region, however, \( \psi = 0 \), i.e. \( \psi \) is independent of time. When many modes are present the phenomenon may be observed in the time domain as a train of well-defined short pulses (see e.g. [6]).

1. Pressure Broadening

From the preceding section it should be clear that the frequency range around the line center in which locking will occur depends critically on the laser intensity (number of quanta) and on the homogeneous linewidth. Since laser action in a He-Ne mixture can only be obtained at He pressures up from about 0.4 torr and most tubes operate at several torr, the effect of pressure broadening may not be neglected [3]. As Szöke and Javan [7] already indicated, this effect cannot be described by simply increasing the decay rates \( \gamma_a, \gamma_b \) of the atomic levels. Neglecting inelastic processes such as transitions between magnetic sublevels and resonant interactions the following collision model may be considered.

A distinction is drawn between collisions which result in a change of the velocity of the radiating atom (and in essence give rise to amplitude modulation) and collisions which modulate the transition frequency itself.

A) In the case of velocity changing collisions one can think of deflections experienced by an active atom which lead to another velocity along the optical axis such that afterwards the atom radiates at another position under the Doppler profile. Quite generally one may assume that the collisions constitute a Markoff process.

Since all atoms are identical this means in fact that a particular atom will not be recognized after a collision, and the main result is an increase of the decay rates of the active atomic levels. In this “loss of all memory” limit the velocity changing collisions are similar to the “hard collisions”, as described by Szöke and Javan [7]. In fact, their “soft collisions” are not included here, but have the same phenomenological effect as the phase changing collisions, to be discussed next.

B) In the case of phase changing collisions one can separate, following Gyorffy et al. [8], the effects of close and distant collisions.

The phase modulation due to close collisions appears to have a real and an imaginary part, resulting in a shift and a broadening of the transition frequency. Estimates for the shift range from 2 MHz/torr [9] to about 20 MHz/torr [8, 10–12]. In this study it