lines, to the rectangular lattice. Each curve is constructed for a definite value of the parameter $\lambda^* = b^*/a^*$. All the series of curves are constructed with respect to the parameter $\lambda = 2a^*/\omega_1$ ($\omega_1$ is the distance between the centers of the apertures along the axis $OX$).

The curves in Figs. 2-4 show the ratios of the reduced moduli of elasticity $E_1^*$, $E_2^*$, and $G^*$ to the moduli of elasticity of the material $E_1$, $E_2$, and $G$, respectively, while Fig. 5 shows the variation of the reduced Poisson ratio $\nu^*$.

LITERATURE CITED


CALCULATION OF THE EFFECT OF TRANSIENT TEMPERATURE FIELDS ON THE STRESSED STATE OF THERMOVISCOELASTIC BODIES

V. G. Gromov and A. N. Rumyantsev

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It is known that the viscoelastic properties of many materials depend substantially on temperature, and it is difficult to solve boundary-value problems when this dependence must be taken into consideration [1, 2, 3]. Therefore, it is advisable to distinguish a separate class of problems which admit of mathematical treatment and thermomechanical analysis. Such problems must be primarily those which can be described relatively simply in terms of space variables. This condition is satisfied by one-dimensional and plane problems of thermoviscoelasticity. In what follows, using the example of such problems, we give the calculation of the effect of transient temperature fields on the stressed state of viscoelastic bodies.

1. Fundamental Formulas

We consider a plane deformation of a thermoviscoelastic body subjected to the action of a transient temperature field $T$. For convenience, we shall use in our investigation the complex coordinates

$$z = x_1 + ix_2; \quad \bar{z} = x_1 - ix_2.$$ (1.1)

The displacement field $U = u_1 + iu_2$ arising in the problem may be considered a potential field [5]. In the present case it can be described as follows:

$$U(z, \bar{z}, t) = 2 \frac{\partial}{\partial z} \Phi(z, \bar{z}, t),$$ (1.2)

where $\Phi(z, \bar{z}, t)$ is the displacement potential.
Setting the Poisson coefficient equal to a constant ($\nu = \text{const}$), we reduce the determining equations of thermoviscoelasticity to the form

$$\sigma_{ij} = \frac{1}{1+\nu} \tilde{E} \left( \varepsilon_{ij} + \frac{1}{1-2\nu} [\nu \varepsilon_{kh} - (1+\nu) \alpha (T-T_0) \delta_{ij}] \right) \quad (i,j = 1,2). \tag{1.3}$$

Here

$$\tilde{E} = E_0 + \int_0^\delta E_T(\lambda) \int_0^1 e^{-\lambda \mu_T^{(i)}} D(\cdot) d\lambda d\mu; \tag{1.4}$$

$\tilde{E}$ is the thermoviscoelasticity operator, represented by the distribution function $E_T(\lambda)$ of the nonisothermal spectrum of relaxation times $\mu_T$; $D = d/d\tau$.

Introducing the thermal-effect parameter $\varepsilon$, we can obtain an expansion of the distribution function $E_T$ of the nonisothermal spectrum $\mu_T$ and, consequently, also of the operator (1.4) in a series in powers of this parameter:

$$\tilde{E} = \tilde{E}^0 + \varepsilon \tilde{E}^1 + \ldots \quad (\mu_T^{-1} = \lambda_0 + \varepsilon \lambda_1 \theta + \ldots, \theta = T - T_0), \tag{1.5}$$

where $\tilde{E}^0$ is the isothermal viscoelasticity operator.

For example, for a normal body $E_T(\lambda) = E_0 \delta(\lambda - \lambda_0)$ we have

$$\tilde{E}^0 = E_0 \left( 1 - \delta \theta_0 \int_0^1 e^{-\lambda \mu_T^{(i)}} (\cdot) d\tau \right) \quad (E = E_0 + E_i; \quad b = E_i / \tilde{E}); \tag{1.6}$$

$$\tilde{E}^1 = \int_0^\delta \left( \delta \theta_0 - \theta_0 \int_0^1 e^{-\lambda \mu_T^{(i)}} (\cdot) d\tau \right) e^{\lambda \mu_T^{(i)}} (\cdot) d\tau.$$

With the expansion (1.5) we can apply the method of perturbations when we solve the boundary-value problem. The displacement potential should have the form of a series

$$\Phi(\xi, \eta, \tau) = \sum_{k=0}^\infty \Phi_k e^{\xi \theta} \tag{1.7}$$

To determine the coefficients $\Phi_k$, we obtain a sequence of boundary-value problems. Omitting the intermediate calculations, we give below the final form of the equations for $\Phi_k$ which are obtained from the kinematic relations and the equilibrium equations:

$$\tilde{E} \frac{\partial^2 \Phi_k}{\partial \xi^2} = f_k (\xi, \eta, \tau); \quad f_k = f_k (\Phi_0, \Phi_1, \ldots, \Phi_{k-1}). \tag{1.8}$$

It is a characteristic fact that $f_0 = [(1+\nu)/(1-\nu)] \alpha \tilde{E}^0 \theta$. This enables us to obtain for $\Phi_0$ (making use of the property of the operator $E_0$ that if $\tilde{E}^0 f_0 = 0$, then $f_0 = 0$) the simpler equation

$$\frac{\partial^2 \Phi_0}{\partial \xi^2} = \frac{1+\nu}{4(1-\nu)} \alpha \theta, \tag{1.9}$$

from which it follows that the displacement potential in the principal part is determined only by the temperature and is independent of the viscosity properties. It begins to manifest itself in the first approximation, since

$$f_1 = \frac{1-2\nu}{2(1-\nu)} \left( \int_0^\delta \frac{\partial^2 \Phi_0}{\partial \xi^2} \frac{\partial}{\partial \xi} \left( \int_0^\delta \frac{\partial \Phi_0}{\partial \xi^2} \right) d\xi \right). \tag{1.10}$$

Knowing the potential, we can find the stresses

$$\sigma_{11} = \frac{\tilde{E}^0}{1+\nu} \left( \frac{\partial^2 \Phi_0}{\partial \xi^2} + \frac{\partial^2 \Phi_0}{\partial \eta^2} - 2 \frac{\partial^2 \Phi_0}{\partial \xi \partial \eta} \right); \quad \sigma_{12} = \frac{\tilde{E}^0}{1+\nu} \left( \frac{\partial^2 \Phi_0}{\partial \xi^2} - \frac{\partial^2 \Phi_0}{\partial \eta^2} \right); \tag{1.11}$$

$$\sigma_{22} = \frac{\tilde{E}^0}{1+\nu} \left( \frac{\partial^2 \Phi_0}{\partial \eta^2} + \frac{\partial^2 \Phi_0}{\partial \xi^2} + 2 \frac{\partial^2 \Phi_0}{\partial \xi \partial \eta} \right).$$