From (4.1) and (4.5) we get the desired relationship between the increments in the stress and strain tensors:
\[ \delta \sigma = 2G(\varepsilon^0 - A) \left( I + \frac{k}{1 - k} A \right)^{-1} \delta \varepsilon. \]

The broken line in Fig. 3 shows \( \sigma_i/sup_i \) as derived in the generalized singular approximation. As would be expected, the loading has become somewhat milder than in the previous case.

LITERATURE CITED


NECKING IN STRETCHING OF A STRIP WITH TWO-DIMENSIONAL STRAIN

G. D. Del' and S. S. Oding

A study has been made [4] of the development of a neck in the stretching of a strip of an ideally plastic material. The linearized plasticity condition was used as the plastic potential. This made it possible to obtain a simple solution that ruled out curvature in the characteristics, which conflicts with the conditions at the outer edge of the neck.

Here we linearize the relationships on the characteristics. The solution allows one to construct the distribution of the velocity and stress in the region of the neck.

We consider the stretching along the x axis for a strip of thickness 2h made of ideally plastic material (Fig. 1). In the subcritical state (i.e., before the formation of a neck) the stresses are \( \sigma_x = \tau_y = \tau_z = \sigma_z = 0 \). We consider the deformation planar and have a strain rate \( \varepsilon_z = 0 \) and \( K = \sigma_z = \sigma_x/2 \), where the hydrostatic pressure is \( \sigma^0 = K \).

We introduce a system of coordinates \( \alpha \) and \( \mu \) coincident with the slip lines in the subcritical state (Fig. 1). The inclination of the tangent to a slip line is \( \theta^0 = -\pi/4 \).

The components of the state of strain satisfy the system
\[ \sigma_x = \sigma - K \sin 2\alpha; \quad \sigma_y = \sigma + K \sin 2\alpha; \quad \tau_{x\mu} = K \cos 2\alpha. \] (1)

The relations on the slip lines take the form
\[ \sigma = 2K\theta = A_{\alpha} \text{ on an } \alpha \text{ line; } \]
where \( A_\alpha, A_\beta \) are constant along the \( \alpha \) and \( \beta \) lines, respectively.

Along with uniform stretching we consider stretching of a strip with a certain fairly small deviation from regular form in the shape of a neck. The components of the state of stress and strain differ from the initial ones by infinitely small amounts denoted by \( \delta \).

Linearization of (1) and (2) gives

\[\delta \sigma_x = \delta \sigma; \quad \delta \sigma_y = \delta \sigma; \quad \delta \tau_{xy} = 2K \delta \theta; \quad \delta \sigma - 2K \delta \theta = \delta A_\alpha \text{ on an } \alpha \text{ line,} \]
\[\delta \sigma + 2K \delta \theta = \delta A_\beta \text{ on a } \beta \text{ line.} \]

Here \( \delta A_\alpha \) and \( \delta A_\beta \) are constants on the \( \alpha \) and \( \beta \) line, respectively.

Let the shape of the neck be described in dimensionless form by the smooth curve

\[ \frac{y}{h} = \delta \varphi \left( \frac{x}{h} \right). \]

Here and subsequently, the coordinates \( x, y, \alpha, \) and \( \beta \) are considered dimensionless, i.e., are referred to \( h; \varphi(0) = -1; \varphi(0) = 0; \varphi(-x) = \varphi(x). \)

The solution in the region of the neck is defined in the triangle \( OA'A \) (Fig. 1). The lines \( OA' \) and \( OA \) are lines of discontinuity in the stress and velocity.

The boundary conditions in terms of the stress may be put as follows: on \( AA' \)

\[ \sigma_v = \sigma - K \sin 2(\theta - \psi) = 0; \quad \tau_v = K \cos 2(\theta - \psi) = 0. \]

Here \( \psi \) is the angle between the normal to the edge of the neck \( v \) and the \( x \) axis.

Linearization of (6) gives us as follows: on \( A'A \)

\[ \delta \sigma = 0; \quad \delta \theta = \delta \psi = \delta \cdot \varphi'(x). \]

On the symmetry axis \( OO' \) we have

\[ \delta \theta = 0. \]

Using (4) with boundary conditions (7) and (8) we determine the components \( \delta \sigma \) and \( \delta \theta \) at some point \( B \) belonging to triangle \( OA'A \) (Fig. 2); from (4) we get

\[ \delta \sigma_B - \delta \sigma_C + 2K (\delta \theta_B - \delta \theta_C) = 0; \quad \delta \sigma_B - \delta \sigma_D - 2K (\delta \theta_B - \delta \theta_D) = 0. \]

We solve this system on the basis of (7), namely \( \delta \sigma_C = \delta \sigma_D = 0, \) which gives

\[ \delta \theta_B = \frac{1}{2} (\delta \theta_C + \delta \theta_D); \quad \delta \sigma_B = K (\delta \theta_C - \delta \theta_D). \]

In the \( \alpha - \beta \) coordinate system we have found from (7) and the fact that \( x = \sqrt{2} \alpha = \sqrt{2} \beta \) on \( A'A \) that

\[ \delta \theta (\alpha, \beta) = \delta \frac{1}{2} \left( \psi' (\sqrt{2} \alpha) + \psi' (\sqrt{2} \beta) \right); \quad \delta \sigma (\alpha, \beta) = K \delta \left[ \psi' (\sqrt{2} \alpha) - \psi' (\sqrt{2} \beta) \right]. \]