Consider the contact of an elastic half-plane with a heated region of finite size, made of a material with the properties of creep and aging. It is assumed that the behavior of this material is described by equations of the creep theory of aging bodies [1, 2]. As a preliminary, a variational equation of the given problem is obtained, expressing the condition of a maximum of some thermodynamic potential associated with the Gibbs thermodynamic potential. Problems of this type are of interest in hydroengineering in connection with the need to calculate the thermostress state of concrete plates and blocks placed on a base.

1. Consider a piecewise-homogeneous isotropic plane-deformed region \( \Omega \), consisting of a region of finite size \( \Omega^{(1)} \) and a half-plane continuously joined to it \( \Omega^{(2)} \) (Fig. 1). The material of region \( \Omega^{(1)} \) has the properties of creep and aging; the half-plane is elastic. The boundary \( S^{(1)} \) of region \( \Omega^{(1)} \) and boundary \( S^{(2)} \) of the half-plane \( \Omega^{(2)} \) have a common rectilinear section \( S \). The Cauchy equation of quasistatic equilibrium and the boundary conditions at time \( t \) take the form

\[
\varepsilon_{ij}^{(b)}(t) = 2^{-1} [u_{ij}^{(b)}(t) + u_{ij}^{(b)}(t)]; \quad \sigma_{ij}^{(b)}(t) = 0 \text{ in } \Omega^{(b)}; \quad \sigma_{ij}^{(b)}(t) = 0 \text{ on } S^{(b)} \setminus S \quad (\beta = 1, 2). \tag{1.1}
\]

Here and below, \( i, j = 1, 2; \varepsilon_{ij}^{(b)}(t), \sigma_{ij}^{(b)}(t), u_{ij}^{(b)}(t) \) are components of the strain and stress tensors and the displacement vector, respectively.

In addition, the condition that the stresses \( \sigma_{12}^{(1)} \) and \( \sigma_{12}^{(2)} \) must be equal, and likewise \( \sigma_{22}^{(1)} \) and \( \sigma_{22}^{(2)} \), on the section \( S \), the same condition for the displacements \( u_{11}^{(1)} \) and \( u_{11}^{(2)} \) and also \( u_{12}^{(1)} \) and \( u_{12}^{(2)} \), and the condition of damping of the stress at infinity in the half-plane \( \Omega^{(2)} \) must be satisfied. The components of the stress and strain tensors are related as follows:

\[
\varepsilon_{ij}^{(0)}(t) = (1 + v^{(0)}) \left[ (U + L) \left( \sigma_{ij}^{(0)}(t) - \varepsilon_{ij} (U + L) \frac{\sigma_{kk}^{(0)}(t)}{E^{(0)}} \right) + \delta_{ij} \varepsilon^{(0)}(t) \right]; \tag{1.2}
\]

\[
u_{ij}^{(0)}(t) = (1 + v^{(0)}) \left[ \sigma_{ij}^{(0)}(t) - \varepsilon_{ij} \frac{\sigma_{kk}^{(0)}(t)}{E^{(0)}} + \delta_{ij} \varepsilon^{(0)} \right] \tag{1.3}
\]

where

\[
J_{\varphi} = \varphi(t); \quad L_{\varphi} = \int_{t_0}^{t} \varphi(t) \frac{dP(t, \tau)}{dt} d\tau; \quad P(t, \tau) = -E^{(1)}(t) \frac{\partial}{\partial \tau} \delta(t, \tau); \quad \delta(t, \tau) = 1/E^{(1)}(t) + C(t, \tau); \quad \varepsilon^{(0)}(t) = T^{(0)}(t) - T^{(0)} \quad T^{(0)} = T^{(0)}(t); \quad T^{(0)} = T^{(0)}(t)
\]
Fig. 1

t₀ is the time at which temperature perturbations are first applied; $E^{(1)}(t)$ is the modulus of instantaneous elastic strains; $E^{(2)}$ is the modulus of elasticity; $\nu(\beta)$ is Poisson's ratio; $\alpha(\beta)$ is the linear-expansion coefficient; $C(t, \gamma)$ is a measure of the creep; $\delta_{ij}$ is the Kronecker delta.

2. In [8], an expression was obtained for the free-energy density of a body with the properties of creep and aging; the equation of state takes the form in Eq. (1.2). Using the results of [8], the well-known expression for the free-energy density of an elastic body, and the relation between the bulk free-energy densities $w^{(\beta)}(t)$ and the thermodynamic Gibbs potential

$$h^{(\beta)}(t) = w^{(\beta)}(t) - \sigma^{(\beta)}(t) \varepsilon^{(\beta)}(t),$$

and taking into account that the case of plane deformation is being considered, the following relations are obtained

$$h^{(\beta)}(t) = \frac{x^{(\beta)}(1 + x^{(\beta)})}{2E^{(\beta)}} \left[ \sigma^{(\beta)}(t) \right]^2 - \frac{x^{(\beta)}}{2E^{(\beta)}} \sigma^{(\beta)}(t) \sigma^{(\beta)}(t) - \frac{c^{(\beta)}}{1 - 2x^{(\beta)}} \left[ \sigma^{(\beta)}(t) \right]^2 \varepsilon^{(\beta)}(1 + x^{(\beta)}),$$

$$a^{(\beta)}(t) = \frac{c^{(\beta)}}{T^{(\beta)}} \left[ \varepsilon^{(\beta)}(1 + x^{(\beta)}) \right] - \frac{3}{1 - 2x^{(\beta)}} \left[ \sigma^{(\beta)}(t) \right]^2 \varepsilon^{(\beta)}(1 + x^{(\beta)});$$

$$s^{(\beta)}(t) = -\frac{\partial h^{(\beta)}(t)}{\partial x^{(\beta)}(t)}; \quad s^{(\beta)}(t) = \frac{a^{(\beta)}(t)}{T^{(\beta)}(t)}.$$ (2.1)

Here and below, $s^{(\beta)}(t)$ is the entropy density, while

$$c^{(\beta)} = \frac{c_{\text{\(\beta\)}}^{(\beta)} T^{(\beta)}(t)}{T^{(\beta)}(t)} - \frac{3}{1 - 2x^{(\beta)}} \left[ \sigma^{(\beta)}(t) \right]^2 \varepsilon^{(\beta)}(1 + x^{(\beta)});$$

$$c^{(\beta)} = \frac{c_{\text{\(\beta\)}}^{(\beta)} T^{(\beta)}(t)}{T^{(\beta)}(t)} - \frac{3}{1 - 2x^{(\beta)}} \left[ \sigma^{(\beta)}(t) \right]^2 \varepsilon^{(\beta)}(1 + x^{(\beta)});$$

$c_{\text{\(\beta\)}}$ is the bulk specific heat for instantaneous heating and constant stress; the parameter $h^{(\beta)}(t)$ is equal to the density of the potential $h^{(\beta)}(t)$, and the parameter $s^{(\beta)}(t)$ to the density of the entropy $s^{(\beta)}(t)$ for $\sigma^{(\beta)}(t) = 0$.

In view of Eq. (2.1), it follows that

$$h^{(\beta)}(t) = h^{(\beta)}[\sigma^{(\beta)}(t), T^{(\beta)}(t), \gamma^{(\beta)}(t)]; \quad h^{(\beta)}(t) = h^{(\beta)}[\sigma^{(\beta)}(t), T^{(\beta)}(t)].$$ (2.3)

Here $x^{\beta}(t)$ denotes the parameters $E^{(\beta)}(t), L(\gamma^{(\beta)}(t), \gamma^{(\beta)}(t)), c^{(\beta)}(t), s^{(\beta)}(t), h^{(\beta)}(t)$.

Assume that the instantaneous values of the stress $\sigma^{(\beta)}(t)$ are varied, while $(\partial \gamma^{(\beta)}(t)) = 0$ and the temperature $T^{(\beta)}(t)$ and parameters $x^{\beta}(t)$ remain unchanged $(\partial T^{(\beta)}(t) = 0, \partial x^{(\beta)}(t) = 0)$ in $\Omega^{(\beta)}$. Then, taking account of Eqs. (2.3) and (2.2), the Cauchy equation, and the boundary conditions, it is found that