We consider the existence conditions for steady nonoscillatory and oscillatory modes of combustion of a homogeneous mixture fed through one end of a tube of finite length. Unlike in previous studies [1, 3, 4, 7], here the kinetics of the total chemical reactions will be regarded as providing the feedback mechanism.

§ 1. According to the thermal theory, processes of "perturbed" combustion are describable by the following system of nonlinear equations:

\[
\frac{du'}{dt} + \frac{\partial}{\partial z} \left( u' \frac{\partial u'}{\partial z} \right) - \frac{\theta}{x_{0}^{2}} \frac{dp'}{dz} \rho' + \frac{1}{x_{0}^{2}} \frac{\partial}{\partial z} \left( \theta \rho' + \rho \theta' \right) + \\
+ \frac{1}{x} \frac{\partial \theta'}{\partial z} - \frac{4}{3N} \frac{d\eta'}{dz} + \frac{1}{x_{0}^{2}} \frac{dp'}{dz} \rho'' + \frac{1}{x_{0}^{2}} \frac{d\theta'}{dz} \rho' - \\
- \frac{\theta}{x_{0}^{2}} \frac{dp'}{dz} \rho' \theta' + \frac{1}{x_{0}^{2}} \rho' \frac{\partial \theta'}{\partial z} + \frac{1}{x_{0}^{2}} \rho' \frac{\partial \eta'}{\partial z} + \\
+ \frac{1}{x_{0}^{2}} \frac{dp'}{dz} \rho' \theta' + \frac{1}{x_{0}^{2}} \rho' \frac{\partial \theta'}{\partial z} = 0;
\]

\[
\frac{\partial \eta'}{\partial t} + \frac{\partial}{\partial z} \left( \rho \eta' + u \rho' + \rho'u' \right) = 0;
\]

Here \( z \) is the dimensionless coordinate referred to the length of the tube \( l \), \( u \) is the dimensionless velocity component referred to the local velocity of sound \( a_{0} \) at the tube entrance, \( t \) is the dimensionless time referred to the characteristic time \( l/a_{0} \), \( \rho \) is the dimensionless density referred to the density of "cold" gas, \( \theta \) is the dimensionless temperature referred to the initial temperature \( T_{0} \) of the mixture, \( p \) is the dimensionless pressure referred to \( \rho_{0} \), \( Le = \lambda/c_{0} \rho D \) is the Lewis number, \( D \) is the coefficient of diffusion, \( Pr = \frac{\nu}{\sigma_{0}} \) is the Prandtl number, \( \chi \) is the thermal diffusivity, \( u \) is the dynamic viscosity, \( Da = \frac{\sigma_{0} I}{a_{0} c_{0} \rho_{0} T_{0}} \) is the Damkohler number, \( W_{s}(\rho, \theta) = \sum_{i=1}^{\infty} (B-\theta)^{2} p_{i}^{e} - \frac{A_{t}^{*}}{\theta} \) is the rate of a total chemical reaction, \( \alpha = \Sigma_{i=1}^{\infty} \), \( \gamma = \alpha - 1 \), \( Ar = E/RT_{0} \) is the Arrhenius number, \( N = \frac{a_{0} I}{v} \) is the acoustic Reynolds number, and \( Po = \frac{q_{0} I^{2}}{\chi T_{0}} \) is the Pomerantsev number.

The solution to system (1.1) must satisfy the boundary conditions (1.2) and (1.3) in [5].

System (1.1) of partial differential equations will be reduced to a system of ordinary differential equations, for which purpose we expand the solution into finite series.
\[ u'(t, z), p'(t, z), \theta'(t, z) = \sum_{i=1}^{n} T_i^{(k)}(t) \phi_i^{(k)}(z) \quad (k = 1, 2, 3), \]  

where \( \phi_i^{(k)}(z) \) are integrating functions and \( T_i^{(k)}(t) \) are the sought time functions.

The system of equations with respect to \( T_i^{(k)}(t) \) can be constructed from the condition that the standard deviation of the operator of the original differential equation from the zero of the function be minimum on the interval within the variable changes:

\[ \int_{0}^{1} L \left[ \frac{dT_i^{(k)}(t)}{dt}, T_i^{(k)}(t), \phi_i^{(k)}(z) \right] \frac{\partial L}{\partial \left( \frac{dT_i^{(k)}(t)}{dt} \right)} \, dz = 0 \quad (i = 1, 2, \ldots, n). \]

Inserting the expansion (1.2) into the condition (1.3) and performing the integration, we obtain an infinite system of nonlinear equations.

According to the results in [2, 6], the infinite system can be replaced with a system of the kind

\[ \frac{dT^{(1)}}{dt} - p_{12} T^{(1)} = \mu F_1 [T^{(1)}, T^{(2)}, T^{(3)}], \]

\[ \frac{dT^{(2)}}{dt} + p_{23} T^{(1)} = \mu F_2 [T^{(1)}, T^{(2)}, T^{(3)}], \quad \frac{dT^{(3)}}{dt} = \mu F_3 [T^{(1)}, T^{(2)}, T^{(3)}] \]

and the necessary accuracy maintained.

With the substitution

\[ T^{(1)} = \frac{dU(t)}{dt}; \quad T^{(2)} = \frac{2p_{21}}{p_{12}} \frac{dU}{dt} + p_{23} U; \quad T^{(3)} = T(t), \]

system (1.4) becomes

\[ \frac{dU}{dt^2} + \omega^2 U = \mu \Phi_1 (U, T); \quad \frac{dT}{dt} = \mu \Phi_2 (U, T), \]

where \( \omega^2 = -p_{12}/p_{21} \) is the eigenvalue of the corresponding linear problem and \( \Phi_1, \Phi_2 \) are nonlinear functions:

\[ \Phi_1 = -p_{11} \frac{dv}{dt} - p_{12} \frac{dv}{dt} + p_{11} \left( \frac{dv}{dt} \right)^2 + 2p_{21} \frac{dv}{dt} + p_{21} \left( \frac{dv}{dt} \right)^2 - \]

\[ - p_{23} \left( 2p_{21} \frac{dv}{dt} + p_{23} v \right)^2 - p_{32} \left( 2p_{21} \frac{dv}{dt} + p_{23} v \right) - p_{31} \frac{dv}{dt} \left( 2p_{21} \frac{dv}{dt} + p_{23} v \right); \]

\[ \Phi_2 = -p_{31} \frac{dv}{dt} + p_{32} \left( 2p_{21} \frac{dv}{dt} + p_{23} v \right) - p_{31} \frac{dv}{dt} + p_{23} \frac{dv}{dt} - p_{23} \frac{dv}{dt} + p_{23} \frac{dv}{dt}; \]

System (1.6) will be reduced to "standard form" by the change of variables

\[ v = V \cos \psi; \quad \frac{dU}{dt} = -V_0 \sin \psi. \]

Here \( \psi = pt + \varphi \) and \( p \) is the frequency of spontaneous oscillations.

In "standard form" the system of equations is

\[ \frac{dV}{dt} = -\frac{\mu}{\omega} \Phi_1 \sin \psi; \quad \frac{d\psi}{dt} = \omega - \frac{\mu}{V_0} \Phi_1 \cos \psi; \quad \frac{dT}{dt} = \mu \Phi_2. \]

It constitutes an approximate mathematical model describing the most general characteristics of the original system with distributed parameters in this study.

\[ \S 2. \] Following the method of perturbation theory [2], we will seek the approximate solution to the system of equations in the form