attempt to take account of the interaction of the crystallites more precisely at this stage would be unjustified, since the original model is itself fairly imprecise. More significant results may be expected from a consideration of the nonlinear variation of \( \gamma \) as a function of \( \tau \).

**LITERATURE CITED**


**STRESSED STATE OF A PIECEWISE-HOMOGENEOUS PLATE WITH AN ELASTIC INCLUSION**

A. A. Syas'tkii and V. A. Syas'tkii

We present an approximate method for determining the stresses in a piecewise homogeneous isotropic plate with a curvilinear inclusion in the form of a part of a thin elastic rod of constant cross section for given stresses "at infinity." By using the Muskhelishvili method [4, 6] we reduce the problem to a system of four singular integral equations which we solve by the Multopp-Kalandiya method of boundary collocation [1]. We consider special cases of the problem for a piecewise homogeneous plate with an arc cut, and an infinite plate with a partially reinforced circular opening.

1. We consider a piecewise homogeneous system consisting of an infinite plate with a circular opening of radius \( \rho_0 = 1 \) and a circular plate joined to it along the part \([-\alpha_0; \alpha_0]\) by a portion of a thin elastic ring (rod) of constant cross section. The system is acted upon by two perpendicular tensile stresses \( p \) and \( q \) "at infinity." There are no external loads on the joining line or on the free edges which do not make contact in the deformation process.

The middle plane of the plate is referred to a polar coordinate system \((\rho, \lambda)\) with the pole at the center of the circular plate. The polar axis passes through the middle of the reinforcing rod and forms an angle \( \rho_0 \) with the direction of \( p \) (Fig. 1). We regard the reinforcing rod as an elastic line working only in tension and bending in the plane of the plate [7]. Under this assumption the deformations of the plates on the joining line will be identical.

We find the stressed state of the plates on the line of separation of the materials from the formulas [6]

\[
\begin{align*}
\phi^- (t_0) &= \phi^+ (t_0) = f(t_0); \\
\phi^+ (t_0) &= 4\frac{B}{\rho_0^2} \left( X + i\gamma \right); \\
\kappa \phi^+ (t_0) &= 4 \frac{B h_1 (X_1 + i\gamma)}{\rho_0^2 (x_1 - i\gamma)}.
\end{align*}
\]

Here

\[
\begin{align*}
\phi^+ (t_0) &= \pm \frac{1}{2} f(t_0) - \frac{1}{2ni} \int_{t_0}^t \frac{f(t)}{\tau - t} \frac{dt}{\tau}; \\
\phi^- (t_0) &= \pm \frac{1}{2} f(t_0) + \frac{1}{2ni} \int_{t_0}^t \frac{f(t)}{\tau - t} \frac{dt}{\tau}.
\end{align*}
\]
2h_j, G_j, \nu_j, and E_j are respectively the thicknesses of the plates, the shear moduli, Poisson ratios, and the elastic moduli; T^\rho (t) and S^\lambda (t) are the normal and tangential contact stresses on the joining line; \( j = 0, 1 \); the subscript 1 refers to the circular plate; to simplify writing we omit the subscript 0.

Separating the real and imaginary parts in (1.1) and using (1.2) and the relation

\[
\frac{dt}{\tau - t_0} = \left( \frac{1}{2} \frac{1}{\frac{i}{2} \cos \frac{\lambda - t}{2}} \right) dt,
\]

we find on the line of separation of the materials of the plates

\[
X - X_0 = \frac{1}{2E_h} \left( 1 - \nu_1 \right) T^\rho \left( \int_{x_0}^{x_1} \left( S^\lambda (t) \csc \frac{\lambda - t}{2} dt \right) \right) - \frac{1}{E_h} \frac{\partial u^0}{\partial x} \left( \int_{x_0}^{x_1} \left( S^\lambda (t) \csc \frac{\lambda - t}{2} dt \right) \right);
\]

\[
Y - Y_0 = \frac{1}{2E_h} \left( 1 - \nu_1 \right) S^\lambda \left( \int_{x_0}^{x_1} \left( T^\rho (t) \csc \frac{\lambda - t}{2} dt \right) \right) + \frac{1}{E_h} \frac{\partial u^0}{\partial x} \left( \int_{x_0}^{x_1} \left( T^\rho (t) \csc \frac{\lambda - t}{2} dt \right) \right);
\]

\[
X_1 = \frac{1}{2E_h} \left( 1 - \nu_1 \right) \frac{S^\lambda (t) \csc \frac{\lambda - t}{2} dt}{x_0} \left( \int_{x_0}^{x_1} \left( T^\rho (t) \csc \frac{\lambda - t}{2} dt \right) \right);
\]

\[
Y_1 = \frac{1}{2E_h} \left( 1 - \nu_1 \right) \frac{S^\lambda (t) \csc \frac{\lambda - t}{2} dt}{x_0} \left( \int_{x_0}^{x_1} \left( T^\rho (t) \csc \frac{\lambda - t}{2} dt \right) \right),
\]

where

\[
X_0 = \frac{1}{2E_h} \left( \nu + q \right) - 2 \left( \nu - q \right) \cos 2(\lambda + \beta) ;
\]

\[
Y_0 = \frac{1}{E_h} \left( \nu - q \right) \sin 2(\lambda + \beta).
\]

We find the circumferential stresses on the line of separation of the materials from the formulas

\[
T^\lambda = T^\rho + \frac{1}{E_h} \left( \int_{x_0}^{x_1} S^\lambda (t) \csc \frac{\lambda - t}{2} dt \right) - \frac{1}{2 \pi} \left( \int_{x_0}^{x_1} T^\rho (t) dt + 2E_h X_0 \right);
\]

\[
T^{\lambda(1)} = T^{\rho(1)} - \frac{1}{2 \pi} \left( \int_{x_0}^{x_1} S^\lambda (t) \csc \frac{\lambda - t}{2} dt \right).
\]