The problem of the vibrations of plates on point supports has not been adequately studied. Most of the known papers are devoted to an investigation of free vibrations of rectangular and square plates.

Tso [9] uses the Rayleigh – Ritz method to determine the fundamental frequency of a square plate supported at various points on the diagonals, and presents experimental results.

Petyt and Mirza [7] use the finite-element displacement method to find the natural frequencies of square and rectangular plates on point supports. They investigate the effect of the rigidity of the supports and their area of contact with the plate on the frequency of free vibrations. They discuss plates on multiple supports located in various positions, and give a bibliography of papers on the vibrations of rectangular and square plates supported at various points.

Johns and Nataraja [6] discuss a square plate supported at four points on the diagonals or at the midpoints of the edges. They present nodal patterns for various vibrational modes. The calculations are performed by solving the Germain–Lagrange equations by the finite-difference method.

Kuz'mich and Puzikov [3] describe a procedure and present the results of theoretical and experimental studies of the natural vibrational modes of isotropic rectangular plates with hinged supports symmetrically located with respect to the center of the plate.

Southwell [8] examines the free vibrations of a disk clamped at the center, and gives the natural frequencies for the lowest vibrational modes.

Chi [5] analytically determines the natural frequencies and vibrational modes of a circular plate with simple supports at three equally spaced points on the circumference.

In the present article we employ the theoretical–experimental method [4] to study the free vibrations of square, circular, and triangular plates on point supports symmetrically placed with respect to the center.

The main purpose of the present paper is to determine the location of point supports for which the fundamental frequency of a plate is maximum.

§1. Square Plate. We use the Rayleigh method to solve the problem of the free vibrations of a square plate on point supports symmetrically placed at various points on the diagonals. The potential and kinetic energies of deflection of a vibrating isotropic square plate of side 2a have the form [2]

\[ V = \frac{1}{2} \int_{s} D \left[ \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{\partial^2 w}{\partial y^2} \right] + 2\nu \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + 2(1 - \nu) \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \, dx \, dy, \]

\[ T = \frac{1}{2} \int_{s} \frac{\gamma h}{g} \dot{w} \, dx \, dy, \]

where \( s \) is the domain bounded by the periphery of the plate, \( D = \frac{Eh^3}{12(1 - \nu^2)} \) is the flexural rigidity, \( E \) and \( \nu \) are the elastic modulus and Poisson’s ratio, \( h \) is the thickness of the plate, \( \gamma \) is the specific weight, \( g \) is the acceleration due to gravity, \( \dot{w} \) is the time derivative of the deflection expression, and \( x, y \) are coordinates.
We introduce into Eqs. (1.1) the dimensionless coordinates and dimensionless deflection expression
\[ x = x_0, \quad y = y_0; \quad w = \tilde{w}; \quad -1 \leq x_0, \quad y_0 \leq 1 \]
and give the mode shapes as
\[ \tilde{w}(x_0, y_0) = w_n(x_0, y_0) \cos \omega_n \]
where \( w_n(x_0, y_0) \) is a function satisfying the conditions at the supports at the points \((\pm x_0/a, \pm y_0/a)\) and the conditions on the periphery of the plate.

As a result we obtain the following expressions for the maximum values of \( V \) and \( T \):
\[
V_{\text{max}} = \frac{D h}{2a^2} \int \int \left[ (\frac{\partial^2 w}{\partial x_0^2})^2 + (\frac{\partial^2 w}{\partial y_0^2})^2 + 2v \left( \frac{\partial^2 w}{\partial x_0 \partial y_0} \right)^2 \right] dx_0 dy_0;
\]
\[
T_{\text{max}} = \omega_n^2 \frac{\pi a^2}{2\gamma} \int \int w_n^2 dx_0 dy_0. \tag{1.2}
\]
Equating \( V_{\text{max}} \) and \( T_{\text{max}} \) we find
\[
\omega_n = \frac{1}{\int \int w_n^2 dx_0 dy_0} \int \int \left[ (\frac{\partial^2 w_n}{\partial x_0^2})^2 + (\frac{\partial^2 w_n}{\partial y_0^2})^2 + 2v \left( \frac{\partial^2 w_n}{\partial x_0 \partial y_0} \right)^2 \right] dx_0 dy_0 \frac{1}{2} \tag{1.3}
\]
or
\[
\tilde{\omega}_n = f_n \left( \frac{x_0}{a}, \frac{y_0}{a}, \nu \right). \tag{1.4}
\]
Here \( \tilde{\omega}_n = \omega_n (2a)^2 \sqrt{\gamma h / D} \), and \( n \) denotes the vibrational mode number, with \( n = 0 \) corresponding to the fundamental.

For a square plate \( x_0/a = y_0/a \), and Eq. (1.4) takes the form
\[
\tilde{\omega}_n = f_n \left( \frac{x_0}{a}, \frac{y_0}{a}, \nu \right). \tag{1.5}
\]
For a fixed \( \nu \) the function \( f_n \) depends on the single parameter \( x_0/a \) and is easily constructed from experiment.

Figure 1 shows the function \( f_0 \) for the fundamental of a square plate on four supports symmetrically placed on the diagonals. The circles denote our experimental results and the triangles, those reported in [9]; the crosses correspond to theoretical data of [7].

Circular Plate. To determine the natural vibrational frequencies of a circular plate on point supports we use the Rayleigh method as we did for a square plate.

The potential and kinetic energies of deflection of a circular plate of radius \( a \) in polar coordinates have the form [2]
\[
V = \frac{D}{2} \int \int \left[ (\frac{\partial^2 w}{\partial r^2})^2 + \frac{1}{r} \left( \frac{\partial w}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial^2 w}{\partial \theta^2} \right)^2 \right] - 2(1-v) \left( \frac{\partial^2 w}{\partial r \partial \theta} \right)^2 dr d\theta;
\]
\[
T = \frac{1}{2} \int \int \frac{\gamma h}{g} \tilde{w}^2 r dr d\theta. \tag{1.6}
\]
We introduce into Eqs. (1.6) the dimensionless coordinate \( \rho = r/a \) and the dimensionless deflection expression \( w = w/h \) and give the mode shapes as
\[ \tilde{w}(\rho, \theta) = w_n(\rho, \theta) \cos \omega_n \]
Here \( w_n \) is a function satisfying the conditions at the supports with coordinates \((\rho_0, 0), (\rho_0, \theta_0), (\rho_0, 2\theta_0)\) and the conditions on the periphery of the plate.