STRESS STATE OF A COILED FOUR-LAYER CYLINDRICAL SHELL

L. A. Il'in and N. A. Lobkova

We will examine an infinite thin elastic four-layer coiled cylindrical shell, in which each edge of the coil is fastened through two layers, with loading by internal pressure. Such fastening corresponds to deeper welding during the manufacture of coiled welded pipe. However, all of the layers of the shell are closed, and its stress state is different from that created in a shell with concentric layers.

The studies [1-3, 5-8] examined the stress-strain state of coiled cylindrical shells in which each edge of the coil was fastened by one layer. A five-layer coiled shell in which the coil edges were fastened to two layers with a slight overlap was examined in [4]. Here, we study the stress-strain state of a four-layer coiled shell in which the fastening depends on the friction coefficient and on the overlap of the coil edges for the range of from zero to a complete turn — when the shell becomes a five-layer shell. The study was conducted on the basis of the theoretical scheme in [1, 2], in which the shell was assumed to be momentless. We also assume the absence of initial stresses and of gaps and contact compliance between the layers.

A necessary (but not sufficient) condition for slip of the layers is satisfaction of the equation

\[ \left| \frac{\tau}{\sigma} \right| = f, \]  

(1)

where \( \tau \) and \( \sigma \) are the tangential and normal contact stresses, respectively; \( f \) is the friction coefficient.

Equation (1) is also satisfied on the contact surfaces of the layers on the outside of the slip zone. These regions are zones of potential slip: The contacting layers are in an unstable state, and, as soon as the edge of the slip zone comes into contact with the edge of a potential slip zone, the latter immediately becomes part of the slip zone.

The slip zones in the coil are located differently, depending on the value of the friction coefficient \( f \) and the angle of overlap \( \alpha \). The sequence of their change with a decrease in \( f \) is shown in Fig. 1, where the slip zones are represented by bold arcs. Their position is characterized by the angles \( \varphi_{ab}, \varphi_{cd}, \varphi_{bc}, \varphi_{bd} \). The rays \( \alpha, b, c, d, A, \) and \( B \) and the slip zones of the shell cross section are broken down into sections in which the layers work together.

Case I corresponds to large values of \( f \). The slip zones \( ab \) and \( cd \) are located in the region of attachment of edges \( A \) and \( B \) and do not overlap each other. Their presence is due to the fact that the forces and, hence, the strains in the layers to which the edges are fastened undergo a discontinuity at points \( A \) and \( B \) (the same force is distributed over two layers on one side of these points and over three layers on the other side of the points), while such discontinuities cannot exist in the layers adjacent to these layers.

A decrease in \( f \) is accompanied by an increase in the size of the slip zones. At a certain value of \( f \) and depending on the value of \( \alpha \), either rays \( b \) and \( c \) coincide and the slip pattern takes the form \( \Pi \) (the zone of potential slip \( CB \) is immediately incorporated in the slip zone) or rays \( \alpha \) and \( d \) coincide and the slip pattern takes the form \( \Pi' \).

A further increase in \( f \) leads from case \( II \) to case \( III \) if the rays \( c \) and \( A \) coincide or to case \( III' \) if \( \alpha \) and \( d \) coincide. A decrease in \( f \) leads from case \( II' \) to case \( III' \), when rays \( b \) and \( c \) coincide.


46 0038-5298/86/2201-0046$12.50 © 1986 Plenum Publishing Corporation
A subsequent decrease in $f$ from case III, when $a$ and $d$ coincide, and case III', when $c$ and $A$ coincide, leads to case IV corresponding to complete slip in the coil (except, of course, for those layers which are rigidly connected with the fastening of edges $A$ and $B$).

The circumferential forces in the layers are represented by the following formulas [1]:

in cross sections without slip

$$ N = \frac{R_p}{m}; \quad (2) $$

in layers outside the slip zone or between slip zones

$$ N = C e^{-f\varphi}; \quad (3) $$

in layers inside the innermost slip zone

$$ N = C e^{-f\varphi} + \frac{R_p}{k}. \quad (4) $$

Here, $m$ is the number of layers in the cross section; $k$ is the number of layers inside the innermost slip zone; $C$ represents constants subject to determination; $\varphi$ is the polar angle reckoned for each section from its edge in the clockwise direction.

To determine the angles characterizing the position of the slip zones and the constants in the expressions for the forces in the layers for each case of slip, we construct equations which express the condition of compatibility of the work done by the sections into which the shell cross section was broken down by slip zones and by the rays $a$, $b$, $c$, $d$, $A$, and $B$ [1]. These conditions are as follows: 1) general equilibrium (the sum of the forces in the layers in any radial section is equal to $R_p$); 2) equilibrium fastening of edges $A$ and $B$ (the sum of the forces in the two layers on one side of the fastening is equal to the sum of the forces in the three layers on the other side of the fastening); 3) compatibility of the strains on the edge of the slip zone (equality of the strains (forces) in the contacting layers on the edge of the zone); and, 4) compatibility of the displacements of the sections in contact in the slip zone (equality of the elongations of these sections).