Determination of the Critical Parameters of Short-Range Loads for Elastic Systems

I. Ya. Amiro

UDC 539.3

Until now the solution of a number of problems has been obtained for the buckling of elastic systems subjected to pulse loads that vary in time according to different laws. These solutions are based on an assumption of initial deflections in the majority of cases. A sufficiently detailed exposition of the widespread approaches to determining the critical dynamic load is presented in [2, 3], where an exposition of the results of solving certain problems of this kind can be found. Here, within the framework of a linear problem, determination of the critical parameters of a short-range load of the type of an impulse of parabolic outline is constructed on the utilization of an analytical criterion analogous to that proposed in [1] for a triangular pulse.

1. FORMULATION OF THE PROBLEM AND FUNDAMENTAL STATEMENTS

In an elastic system of the type of a rod, plate, or shell subjected to an external load pulse, let compressive, uniformly distributed stresses occur that vary in time according to the law \( \sigma = ct - c_1 t^2 \). It is here assumed that the active time of the load is limited and equal to \( t_0 = c/c_1 \), while the wave nature of the force propagation can be neglected. If the quantities \( c \) and \( c_1 \) are given, then the maximal value of the compressive stresses \( \sigma_{\text{max}} = c^2/4c_1 \) is determined by the expression presented. The coefficients \( c = 4\sigma_{\text{max}}/t_0 \) and \( c_1 = 4\sigma_{\text{max}}/t_0^2 \) can be determined by the active time of the compressive stresses and their maximal values.

The process of buckling of the system under consideration is characterized by its deviation from the position corresponding to the membrane subcritical state.

The problem is to determine those combinations of parameters characterizing the pulse load (\( c \) and \( c_1 \) or \( \sigma_{\text{max}} \) and \( t_0 \)) for which intensive development of deflections becomes possible. Description of the deflection development process on the basis of known methods reduces to considering an ordinary differential equation in the time function \( f_n \) that corresponds to a definite buckling mode (see [1], e.g.)
where \( \sigma_n \) is the critical stress of static buckling in a given mode, and \( \omega_n \) is the free vibrations frequency of a system in the same mode.

Equation (1.1) determines the nature of the possible system motion. It is seen from this equation that although \( \sigma = \sigma_0 - \sigma_1 t^2 < \sigma_n \), the system motion due to the initial conditions is vibrational in nature; when \( \sigma = \sigma_0 - \sigma_1 t^2 \) becomes greater than \( \sigma_n \), the change in displacements corresponding to the buckling mode under consideration will be monotonic in nature. From the viewpoint of studying the stability of elastic systems, an analysis of precisely this monotonic process of strain development is of interest. Consequently, the solutions of (1.1) are examined below only for those buckling modes for which \( \sigma_n < \sigma_{\text{max}} \).

Going over to dimensionless parameters, we introduce the following notation \( t = \frac{t^*}{\omega_0}; \quad c = c^* \sigma_0 \omega_0; \quad c_1 = c_1^* \sigma_0 \omega_0^2 \) (\( \sigma_0 \) is the minimal value of the critical static buckling stresses, while \( \omega_0 \) is the free vibrations frequency of the unloaded system in a mode corresponding to these stresses). Then (1.1) becomes

\[
\frac{d^2 f_n}{dt^*^2} + \omega_n^2 \left( 1 - \frac{\sigma_0}{\sigma_n} \right) f_n = 0.
\]  

The dimensionless time needed so that the compressive stresses \( \sigma = \sigma_0 (c^* t^* - c_1^* t^*^2) \) reach the value \( \sigma_n \) (it is defined as the smaller root of the quadratic equation \( c_1^* t^*^2 - c^* t^* + \frac{\sigma_n}{\sigma_0} = 0 \)) equals

\[
t_n^* = \frac{c_1^*}{2c_1} \sqrt{\frac{c^*}{4c_1^*} - \frac{\sigma_n}{\sigma_0 c_1^*}}.
\]  

Introducing the substitution \( t^* = t^*_{n} + t^*_{\Delta} \), we can reduce (1.2) to the form

\[
\frac{d^2 f_n}{dt^*^2} - (\beta_n t^* - \gamma_n t^*^3) f_n = 0,
\]  

where

\[
\beta_n = \frac{2 \sigma_0^2 \sigma_1}{\omega_0^2 \sigma_n} \sqrt{\frac{c^*}{4c_1^*} - \frac{\sigma_n}{\sigma_0 c_1^*}}; \quad \gamma_n = \frac{\sigma_0^2}{\omega_0^2 \sigma_n} c_1^*.
\]

2. DYNAMICAL INSTABILITY CRITERION

To formulate the analytical dynamical buckling criterion, it is convenient to represent the solution of (1.4) in the form of power series

\[
f_n = A_0 + A_1 t^*_{n} + A_2 t^*_{n}^2 + \ldots.
\]  

Substituting (2.1) into (1.4) and equating the sum of the coefficients for identical powers of \( t^*_{n} \) to zero, all the constants \( A_s \) (\( s \geq 2 \)) can be expressed in terms of \( A_0 \) and \( A_1 \), which are determined from the initial conditions (for \( t^*_{n} = 0 \)), where \( A_0 \) equals the initial amplitude, while \( A_1 \) is the initial motion velocity. If the motion is caused by an initial deflection of the system, then the development of the displacements occurs in conformity with the expression

\[
f_n = A_0 \left[ 1 + \frac{\beta_n}{6} (t^*_{n} - \gamma_n t^*_{n}^3) + \ldots \right].
\]  

The last terms in the square brackets, which are of more complex form, are not presented here since they are not needed later.

For \( c_1 = 0 \), \( \gamma_n = 0 \) the expression (2.2) agrees with the corresponding solution obtained in [1] for the case of a triangular pulse (\( \sigma = \sigma_0 t \)). There, since the cube of the quantity \( t^*_{n} \) starts to grow strongly after it has achieved the value one, the condition \( \beta_n t^*_{n}^3 = 1 \) is taken as the dynamical stability criterion governing the possibility of the beginning of intensive development of deflections, on which base the "critical" time of load action and the corresponding magnitude of the maximal compressive stresses are determined. Here, by analogy, the condition according to which the equality

\[\text{...}\]