The propagation of high-frequency surface and normal waves is of great theoretical and practical interest in connection with the increasing use of these types of waves in surface defectoscopy, acoustic and electronic signal processing, in the study of the physical properties of solids, etc. [3-5, for example].

The propagation of harmonic waves in materials with a regular structure occupies a special place in this field because of the uniqueness of such materials with regard to wave propagation and structural applications [2, 14, 15, and so on]. This problem has been intensively worked out in the mechanics of deformable solids and used to study the dynamical deformation of composites [1, 7, 8, 14, and so on]. Rayleigh and Lamb waves were studied in [12, 14] for isotropic, regularly layered materials. The anisotropy of many materials significantly complicates earlier methods used to treat the problem, particularly numerical calculation. In the present paper the approach of [9] is used to obtain general dispersion relations for Rayleigh and Lamb waves in orthotropic stratified periodic structures for the case when the sagittal plane is a plane of symmetry. In this case the original boundary-value problem reduces to a set of Cauchy problems and therefore well-known and well-tested numerical methods of solution can be applied.

We consider first an unbounded orthotropic medium $\mathbb{R}^3$ which is periodic along the $x_3$ coordinate with period $h$. Let the functions describing the mechanical properties of the medium be finite on the interval of periodicity (they are actually allowed to be infinite but must be absolutely integrable). The propagation of harmonic waves in such a medium in the direction of constant elastic characteristics is described by the system of equations (5) of [9]. If the sagittal plane (here the plane passing through the wave vector and normal to the surfaces of constant elastic properties) is also a plane of symmetry (for example $k_1 = k; k_2 = 0$ in [9]) then this system of equations splits up into two independent subsystems describing the motion for out of plane and plane deformations. Hence in the case of plane deformations of interest to us

$$\{u_1, u_2, \sigma_{13}, \sigma_{23}\} = \{i\eta_1(x_3), \eta_2(x_3), \eta_3(x_3), q_3(x_3)\} \exp(ikx_1 - i\omega t)$$

we have the Hamiltonian system of ordinary differential equations

$$\frac{d}{dx_3} \begin{bmatrix} q \\ p \end{bmatrix} = A(x_3) \begin{bmatrix} q \\ p \end{bmatrix}; \quad A(x_3) = \begin{bmatrix} 0 & R(x_3) \\ -R(x_3) & 0 \end{bmatrix},$$

in which the vectors $q$ and $p$ and the symmetric matrices $R(x_3)$ and $P(x_3)$ have the following structure:

$$q = \text{col}(q_1, q_3); \quad p = \text{col}(p_1, p_3);$$

$$R_{11} = G_{11}^{-1}(x_3); \quad R_{12} = R_{21} = k; \quad R_{22} = \omega^2\rho(x_3);$$

$$P_{11} = -[\lambda_{11}(x_3) - \lambda_{33}^{-1}(x_3) \lambda_{33}^{-1}(x_3)]k^2 - \omega^2\rho(x_3);$$

$$P_{12} = P_{21} = \lambda_{13}(x_3) \lambda_{33}^{-1}(x_3) k; \quad P_{22} = \lambda_{33}^{-1}(x_3).$$

Here $u_1, u_3$ are the components of the displacement vector; $\sigma_{13}, \sigma_{33}$ are components of the stress tensor; $\lambda_{ij}(x_3) = \lambda_{ij}(x_3 + h)$ are the elastic parameters; $\rho(x_3) = \rho(x_3 + h)$ is the density; $\omega$ is the frequency; $k$ is the wave number in the $x_1$ direction.
The \(x_3\) axis is divided into intervals corresponding to the spatial period of the matrix \(A(x_3)\) and the interval \((n-1)h < x_3 < nh\) \((n = 0, 1, \ldots)\) is called the \(n\)-th interval.

The general solution of the system (1) in the \(n\)-th interval can be written in the form

\[
\begin{bmatrix}
q(x_3 + nh - h) \\
p(x_3 + nh - h)
\end{bmatrix} = \sum_{\ell=1}^{4} K_{\ell} \chi_{\ell}^{*} U(x_3) Z_{\ell} \quad (0 \leq x_3 \leq h),
\]  

\[\tag{2}\]

where the \(K_{\ell}\) are undetermined constants; \(U(x_3)\) is the solution matrix of the system (1) such that \(U(0) = I\), where \(I\) is the unit matrix; \(\chi_{\ell}, Z_{\ell}\) are the characteristic numbers, arranged in increasing order, and the corresponding eigenvectors of the matrix \(U(h)\). From the requirement that the solution be bounded as \(x_3 \to +\infty\) we have the conditions

\[
\text{Im} b_j = 0; \quad |\text{Re} b_j| \leq 1 \quad (j = 1, 2),
\]  

\[\tag{3}\]

where \(b_j\) are the solutions of the quadratic equation \(4b^2 - 2d_1b + d_2 - 2 = 0\) obtained by the substitution \(x + \chi_{\ell} = 2b\) from the characteristic equation \(\det[U(h) - \chi I] = 0\) for \(\chi\) (its recursive nature is taken into account [9]), while the quantities \(d_1\) and \(d_2\) are given in terms of the elements of the matrix \(U(h)\).

The conditions (3), where \(b_j = b_j(\omega, k)\), are dispersion relations implicitly relating the frequency \(\omega\) and wave number \(k\) and they determine the passing bands of plane-polarized bulk waves in the orthotropic periodic medium. The relations (3) can be computed numerically if the elements of the matrix \(U(h)\) are known. These elements can be found by solving (for each step in \(\omega\) and \(k\)) four successive Cauchy problems for the system (1) on the interval \(0 \leq x_3 \leq h\) with the following initial conditions:

\[
\begin{bmatrix}
q(0), p(0) \\
q(0), p(0) \\
q(D), p(0) \\
q(0), p(0)
\end{bmatrix} = \begin{bmatrix} 1, 0, 0, 0 \\
0, 1, 0, 0 \\
0, 0, 1, 0 \\
0, 0, 0, 1 \end{bmatrix}
\]  

\[
\text{(the solution of each problem determines a column of the matrix \(U(h)\), since \(U(0) = I\)).}
\]

Figure 1 shows the results of the numerical analysis (1 - \(|b_1| < 1\) \(\wedge\) \(|b_2| < 1\); 2 - \(|b_1| < 1\) \(\wedge\) \(|b_2| < 1\) \(\wedge\) \(|b_3| < 1\) \(\wedge\) \(|b_4| < 1\)); 3 - \(\text{Im} b_j = 0\)), obtained using the Runge-Kutta-Felberg method [13], to the fourth and fifth orders of accuracy (comparison of the two sets of calculated quantities can be used to obtain an estimate of the error, which is necessary to control the stepsize in \(x_3\)). Calculations were done for a piecewise-homogeneous medium formed by alternating two layers of thicknesses \(h_1\) and \(h_2\) \((h_1 + h_2 = h)\) with the following parameters:

\[
\begin{align*}
56\lambda_{11,1}^{-1} &= 56\lambda_{33,1}^{-1} = 30\lambda_{11,3}^{-1} = 13\lambda_{33,3}^{-1} = 48\lambda_{11,5}^{-1} = 48\lambda_{33,5}^{-1} = 95\lambda_{33,3}^{-1} = 37\lambda_{33,5}^{-1} = 189\lambda_{33,5}^{-1}; \\
3\rho_1 &= 2\rho_2; \quad 7h_1 = 3h_2.
\end{align*}
\]

The dimensionless frequency \(\tilde{\omega} = \omega(\rho_1/\lambda_{33,1})^{1/2}\) is plotted on the vertical axis and the dimensionless wave number \(\tilde{k} = k h\) is plotted on the horizontal axis. The solid curves