DETERMINING THE CRITICAL STRESSES IN BENDING OF
A CANTILEVER BEAM WITH A CRACK

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We consider an elastic isotropic strip (beam) of thickness unity, width $2h$, and length $2L$, weakened by a rectilinear slot located in a zone of compressive stresses and directed normal to the axis of the strip (Fig. 1). Let one end of the beam be rigidly clamped and a constant concentrated force $P$ be applied to the other, free end; a uniformly distributed pressure $q$ also acts on the entire length of the beam.

A Cartesian rectangular coordinate system $xOy$ lying in the middle surface of the strip is oriented as shown in Fig. 1. We let $\delta$ be the width of the slot, which is assumed to be commensurate with the elastic displacements.

When the external load reaches a certain value, the lips of the crack meet for a certain length $k_1 \leq x \leq k_2$ (Fig. 2), which produces unknown contact stresses $\sigma(x)$ over this length. The remainder of the slot contour is free of contact stresses.

The problem is solved under the following boundary conditions:

a) for the contact segment $(k_1 \leq x \leq k_2)$,

$$
\sigma_y^+(x, 0) = \sigma_y^-(x, 0); \quad \tau_x^+(x, 0) = q\sigma_y^+(x, 0);
$$

$$
\tau_{xy}^+(x, 0) = q\sigma_y^-(x, 0); \quad v^+(x, 0) - v^-(x, 0) = -\delta,
$$

where $\rho$ is the Coulomb friction coefficient;

b) on the free portions of the slot lips,

$$
\sigma_y^+(x, 0) = \sigma_y^-(x, 0) = 0; \quad \tau_{xy}^+(x, 0) = \tau_{xy}^-(x, 0) = 0.
$$

Using the results of [8], we find that the stress-strain state in a strip with no slot, loaded as shown in Fig. 1, is determined by the function $\Phi(z)$ and $\Omega(z)$:

$$
\Phi(z) = -\frac{q}{120I} [5z^2 - 15(L - d)z^2 + 33h^2 - 5(L - d)^2z + 10h^2] - \frac{P}{8I} [z^2 + 2(L - d)z];
$$

$$
\Omega(z) = -\frac{q}{120I} [35z^2 + 75(L - d)z^2 - 311h^2 + 15(L - d)^3z - 10h^2 - 60h^2(L - d) + \frac{P}{8I} [5z^2 - 6(L - d)z - 4h^2]],
$$

where $I = 2h^3/3$ is the moment of inertia of the strip cross section.

Under the results of [2-4, 6] we obtain formulas for the Muskhelishvili functions $\Phi(z)$ and $\Omega(z)$ for a strip with slot:

$$
\Phi(z) = \frac{1}{2} \left[ \Phi(z) + \Phi(z - \lambda_1) \right] + \frac{Q(z)}{4V(z - a)(z - b)} + \frac{1}{2} [\Phi(z) - \Omega(z)];
$$


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Letting $\lambda_1 = \lambda_2 = \lambda_*$ in (7), we obtain the relationship between the external load $q = q_*$ and the concentrated force $P = P_*$ at which contact between the slot edges occurs.

We now consider the case in which part of the slot contour lies within the region of tensile stresses, while the slot width is zero. Taking $a = -b$, $\delta = 0$, $\lambda_2 = b$, $\rho = 0$ in (5) and (6), we find

$$\Omega(z) = \frac{1}{2} (r_x^2 - r_x^2 + s_a^2 + s_a^2 + s_a^2 + r_s^2) \sqrt{\frac{z - \lambda_1}{z - a}} \frac{\Omega_q(z)}{4 \sqrt{z - a}} + \frac{1}{2} (\Phi_q(z) - \Omega_q(z)).$$

(6)