TRANSVERSE VIBRATIONS OF A SINGLE-SPAN BEAM WITH THE MOTION OF A BENDING MOMENT

A. S. Dmitriev

UDC 624.072.2.042.8.044

The problem of the dynamic action of a moving load on beam-type bridges, trestles, monorail constructions, and special equipment, is of timely importance and, in spite of the great amount of work in this field, continues to attract the constant attention of specialists. Specifically, questions of the dynamics of single-span beams under the action of a vertical moving force have been investigated in [3-5] and, taking account of the mass of the load, in [1, 2, 6, 7].

However, in the general case, in addition to vertical loads, longitudinal horizontal forces are transmitted to the construction, applied at the level of the traveled part of the equipment.

Since, between the level of the application of the moving force displacing the systems, moving in a longitudinal direction, and the line of the centers of gravity of the transverse cross sections of the constructions (the longitudinal axis of the equipment) there is eccentricity, there arises a need to investigate the unstudied question of the vibrations of beams with the motion of a bending moment.

§1. Solution of the Problem without Taking Account of the Inertia of the Load

We consider a single-span statically determinable beam, along which the bending moment \( R \) moves uniformly with the velocity \( v \) (Fig. 1). We introduce the following notation: \( EI \) is the bending rigidity of the beam; \( m \) is the linear mass; \( z(x, t) \) is the deflection of the beam in the cross section \( x \) at the moment of time \( t \).

We expand the dynamic deflections of the beam in series in terms of the forms of the natural vibrations.
where \( f_i(t) \) is the \( i \)-th generalized coordinate.

We represent the moment \( R \) in the form of a pair of vertical forces \( P \), the distance between which is \( \varepsilon \):

\[
R = P\varepsilon.
\]

Then, with the direction of the moment \( R \) shown in Fig. 1, the generalized force \( Q_i \), acting on the beam and equivalent to this moment, has the form

\[
Q_i = P\sin \frac{i\pi t}{l} - P\sin \frac{(vt - \varepsilon)}{l} = 2P\cos \frac{i\pi}{l} (vt - \frac{\varepsilon}{2}) \sin \frac{i\pi \varepsilon}{2l}.
\]

Passing to the limit with \( \varepsilon = 0 \) and \( R = P\varepsilon \), we obtain

\[
Q_i = \frac{inR}{l} \cos \frac{invt}{l}.
\]

Using the results of [3-5], and neglecting resistance forces, we represent the differential equation for the generalized coordinate \( f_i(t) \) in the form

\[
\frac{d^2f_i(t)}{dt^2} + \frac{in^4EI}{m}\frac{df_i(t)}{dt} = \frac{2inR}{ml^2} \cos \frac{invt}{l}.
\]

Introducing the notation

\[
\xi = \frac{vt}{l}; \quad \alpha_i = \frac{in}{l} \sqrt{\frac{EI}{m}},
\]

we have

\[
\frac{d^2f_i(\xi)}{d\xi^2} + \frac{\alpha_i^2}{\alpha_i^2} f_i(\xi) = \frac{2R\alpha_i}{i\pi^2EI} \cos in\xi.
\]

With the initial conditions

\[
f_i(\xi)|_{\xi=0} = \frac{2R\alpha_i}{i\pi^2EI}, \quad \frac{df_i(\xi)}{d\xi}|_{\xi=0} = 0
\]

the solution of Eq. (1.4) is the function

\[
f_i(\xi) = \frac{2R\alpha_i}{i\pi^2EI} \cos \frac{in\xi}{\alpha_i} - \frac{\alpha_i^2 \cos \frac{in\xi}{\alpha_i}}{1 - \alpha_i^2}.
\]

After the passage of the load (\( \xi > 1 \)), the values of \( f_i(\xi) \) are determined by the dependence

\[
f_i(\xi) = \frac{2R\alpha_i}{i\pi^2EI} \frac{(-1)^i \cos \frac{in\xi}{\alpha_i} (\xi - 1) - \alpha_i^2 \cos \frac{in\xi}{\alpha_i}}{1 - \alpha_i^2}.
\]

The deflections of the beam in the cross section \( x \) are calculated using equation (1.1), taking account of the change in the variable. From equations (1.6) and (1.7) it can be seen that, with \( \alpha_i = 1 \), they revert to an indeterminacy of the kind 0/0. In this case, the absolute values of the velocities, called critical, are

\[
v_i = \frac{in}{l} \sqrt{\frac{EI}{m}} (i = 1, 2, \ldots, \infty)
\]