§ 1. We consider the form of steady-state motion in media of different viscosity including gravitational force and the resistive force of the medium with respect to longitudinal motion. Such systems may be encountered in studies of the motion of a ribbon radiator or a flexible radio antenna.

Let an ideal nonexpandible filament (Fig. 1) which runs around the pulley 1 rotating at an angular velocity \( \omega \) move at a velocity \( \omega R \) (\( R \) is the radius of the pulley 1) and pass through two media the boundary between which is at the level \( H \). Depending on the position of the pressure roller 2, the filament may be started out at any angle \( \alpha \) to the vertical. The roller 3, which stabilizes the motion, holds the filament to the pulley so that the filament always encompasses half the circumference of the pulley 1.

The first integrals of the motion have been found [3, 4] for constant viscosity of the medium and the case \( \alpha = \pi /2 \) was discussed in detail. A solution is given [1] in natural coordinates under the same assumptions but for different boundary conditions.

In this paper, a closed solution is obtained for the first time for the boundary-value problem involving a closed filament in steady-state motion through two media. In the system of fixed \( x \) and \( y \) axes, the equations of motion have the form

\[
\frac{d}{ds} \left( T^* \frac{dx}{ds} \right) - \mu \frac{dx}{ds} = 0; \tag{1.1}
\]

\[
\frac{d}{ds} \left( T^* \frac{dy}{ds} \right) - \mu \frac{dy}{ds} - g = 0; \tag{1.2}
\]

\[
\left( \frac{dx}{ds} \right)^2 + \left( \frac{dy}{ds} \right)^2 = 1. \tag{1.3}
\]

Here, \( s \) is the Eulerian arc coordinate; \( T^* = (T/m) - v^2 \); \( T \) is the tension in a filament with a mass per unit length \( m \); \( v \) is the longitudinal velocity of the filament; \( \mu \) is the acceleration due to the resistive force of the medium, which is assumed constant along the filament contour in a given medium; \( g \) is the gravitational constant.

Integration of Eq. (1.1) yields

\[
T^* \frac{dx}{ds} - \mu x = A \tag{1.4}
\]

(\( A \) is a constant of integration).

Substitution of Eq. (1.4) into Eq. (1.2) and conversion from the arc coordinate \( s \) to the Cartesian coordinate \( x \) make it possible to obtain a differential equation with separable variables. Integrating it, we find

\[
y' = -\frac{1}{2} \left( B \left| A + \mu x \right|^k - \frac{1}{B \left| A + \mu x \right|^k} \right), \tag{1.5}
\]

where \( B \) is an arbitrary constant; \( k = g/\mu \); the prime indicates differentiation with respect to \( x \).
After a second integration we have
\[
y = \frac{1}{2} \left( \frac{B |A + \mu x|^{l+x}}{\mu + g} - \frac{|A + \mu x|^{-x}}{B (\mu - g)} \right) + C \quad (1.6)
\]
(C is an arbitrary constant).

In the region under study, the values of the argument are
\[
x < -\frac{A}{\mu} (A < 0); \quad |A + \mu x| = -A - \mu x. \quad (1.7)
\]

It is easy to find the tension in the filament from Eq. (1.4),
\[
T^* = \frac{1}{2} \left( B |A + \mu x|^{l+x} + \frac{|A + \mu x|^{-x}}{B} \right). \quad (1.8)
\]

Equations such as (1.5), (1.6), and (1.8) can be written separately for each of the sections of the filament, for example
\[
y_{ij} = \frac{1}{2} \left( \frac{B_{ij} A_{ij} + \mu_{i} x_{i}^{l+x_{i}}}{\mu_{i} + g} - \frac{|A_{ij} + \mu_{i} x_{i}|^{-x_{i}}}{B_{ij} (\mu_{i} - g)} \right) + C_{ij}. \quad (1.9)
\]

Here, the subscript \( i = 1, 2 \) indicates the portion of the filament contour below and above the level \( H \), respectively, and the subscript \( j = 1, 2 \) refers to the ascending and descending branches.

To determine the constants of integration, we write the following boundary conditions:
\[
y_{ij} \left( -1 \right) R \cos \alpha_{i} = -1, R \sin \alpha_{i}; \quad y_{ij} \left( -\frac{A_{ij}}{\mu_{i}} \right) = y_{22} \left( -\frac{A_{ij}}{\mu_{2}} \right); \quad y'_{ij} \left( -\frac{A_{ij}}{\mu_{i}} \right) = y'_{22} \left( -\frac{A_{ij}}{\mu_{2}} \right) = \infty; \quad y'_{ij} \left( -R \cos \alpha_{i} \right) = \cot \alpha_{i}; \quad (1.10)
\]

\[
y_{ij} (x_{j}) = H; \quad y'_{ij} (x_{j}) = y'_{2j} (x_{j}); \quad T_{ij} (x_{j}) = T_{2j} (x_{j}); \quad \sum_{j} \int V \left( 1 + (y'_{ij})^{2} \right) dx = l
\]

(x_{j} is the abscissa of the points on the filament contour at the boundary between the media).

In the last boundary condition, the total length of the free portion of the filament is \( L \), where the appropriate limits of integration are written for each of the integrals.

Substitution of the boundary conditions yields
\[
(-1)^{l+x} R \sin \alpha_{i} = \frac{1}{2} \left[ \frac{B_{ij} A_{ij} + (-1) R \cos \alpha_{i}^{l+x_{i}}}{\mu_{i} + g} - \frac{|A_{ij} + (-1) R \cos \alpha_{i}|^{-x_{i}}}{B_{ij} (\mu_{i} - g)} \right] + C_{ij}; \quad (1.11)
\]

\[
C_{21} = C_{22} = C_{2} \quad (1.12)
\]