1. Introduction. Contact problems for initially stressed elastic objects undergoing contact interaction with rigid objects with general elastic potentials are presented in [2, 7]. Studies [8, 9] suggest techniques for solving the problem of stress deformation of elastic compressible and incompressible stressed objects undergoing contact interactions with elastic objects.

This study, along with [6, 8, 9], investigates the interaction between a rectangular object of height 2h and width 2a and two stressed half-planes taking into account friction. It is assumed that the initial stresses in the half-planes are formed before contact with the elastic object. We will use the coordinates of the initial deformations state [4, 5, 6], which are related to the Lagrangian coordinates $\chi_i$ (the natural state) by the relations

$$y_i = \lambda_i \chi_i,$$  (1.1)

where $\lambda_i$ is the modulus of elongation, which determines the displacement of the initial state.

We will assume that the initial state is uniform and satisfies the condition

$$S_{ii}^0 = 0; \lambda_1 \neq \lambda_2.$$  (1.2)

2. Presentation of the Problem and the Fundamental Relations. We will consider an elastic rectangle of height 2h and width 2a undergoing compression (strained) by a force applied along the axis $Oy_2$ through the stressed half-planes. The external forces are applied such that points on the line of contact between the elastic rectangle and the half-planes are displaced along axis $Oy_2$ by a quantity $e$, relative to the plane $y_2=0$. The lateral sides of the rectangle-half-plane system external to the line of contact are free from stresses. One must determine the effect which the initial stresses in the half-planes have on the distribution of contact stresses along the line of contact with the elastic rectangle. Similar linear problems (for objects without initial stresses) were considered in [10].

All quantities which pertain to the elastic rectangle will be written in accordance with the definitions of elasticity theory $\sigma_{xx} = \sigma_{yy} = \tau_{xy}$, and those quantities related to the half-planes will be written using the definitions in [6]. Because of the
symmetry of the problem, we will have the following boundary conditions for determining the vector components for the displacements and the stress tensors in the elastic rectangle and the stressed half-planes:

on the free surface of the elastic rectangle \( y_1 = \pm a \)
\[
\sigma_{xx} = 0, \ \tau_{xy} = 0, \quad (2.1)
\]
on the boundary of the elastic half-planes in the contact region
\[
\sigma_{yy} = N(y_1), \ |y_1| \leq a; \quad (2.2)
\]
\[
\tau_{xy} = -T(y_1), \ |y_1| \leq a; \quad (2.3)
\]
\[
u_1 - v = -\varepsilon, \ |y_1| \leq a; \quad (2.4)
\]
\[
u_1 - u = 0, \ |y_1| \leq a; \quad (2.5)
\]
on the boundary of the stressed elastic half-planes at \( y_2 = \pm h \) outside the contact region
\[
\tilde{Q}_{22} = \tilde{Q}_{21} = 0, |y_1| \gg a. \quad (2.6)
\]
Boundary conditions (2.1)-(2.6) along with the equilibrium condition
\[
P = \pi \int_a^b N(t) \, dt \quad (2.7)
\]
give the mathematical presentation of the problem of contact interaction between an elastic rectangle and two identical, stressed half-planes and allow for the study of friction effects.

When one does not take into account friction \( T(y_1) = 0 \), one can also ignore sliding effects (2.5).

The vector components for the displacements \( u_1(y_1, y_2), v(y_1, y_2) \), which satisfy Lamé's homogenous differential equations inside the rectangle \( |y_1| \leq a, |y_2| \leq h \)
\[
\frac{1}{1-2\nu} \frac{\partial^2}{\partial y_1^2} + \Delta u = 0; \quad \frac{1}{1-2\nu} \frac{\partial^2}{\partial y_2^2} + \Delta v = 0 \quad \left( \frac{\partial u}{\partial y_1} + \frac{\partial v}{\partial y_2} = 0 \right), \quad (2.8)
\]
we can find in the form
\[
u_1 = B_0 + \frac{2(1-\nu)}{E} \sum_{\nu=1}^{\infty} \left[ (1-\nu) \frac{F_1'' \nu \nu y_1}{\beta_{\nu}} \right] + (2-\nu) F_1 \nu \nu y_1 \cos \left( \beta_{\nu} y_1 \right) + \frac{2(1-\nu)}{E} \sum_{\nu=1}^{\infty} \left[ (1-\nu) \frac{F_2 \nu \nu y_2}{\alpha_{\nu}} \right] \sin \left( \alpha_{\nu} y_2 \right) + \nu \alpha_{\nu} \nu \nu \nu y_2 \sin \left( \alpha_{\nu} y_2 \right); \quad (2.9)
\]
\[
u_2 = -\frac{2(1+\nu)}{h} \sum_{\nu=1}^{\infty} \left[ (1-\nu) \frac{F_1'' \nu \nu y_1}{\beta_{\nu}} + \nu \beta_{\nu} F_1 \nu \nu y_1 \right] \sin \left( \alpha_{\nu} y_2 \right) + \nu \alpha_{\nu} \nu \nu \nu y_2 \sin \left( \alpha_{\nu} y_2 \right);
\]
\[
u_2 = -\frac{2(1+\nu)}{h} \sum_{\nu=1}^{\infty} \left[ (1-\nu) \frac{F_2'' \nu \nu y_2}{\alpha_{\nu}} - (2-\nu) F_2 \nu \nu y_2 \right] \cos \left( \alpha_{\nu} y_2 \right).
\]

Here, \( \nu, E \) are the Poisson coefficients and the Young modulus of the elastic rectangle, respectively, \( \Delta \) is the Laplace operator, \( \alpha_{\nu} = \pi \nu / h, \beta_{\nu} = \pi \nu / h \), and the functions \( F_1(m, y_1) \) and \( F_2(n, y_2) \) have the form [1, 3]
\[
F_1(m, y_1) = A_m \cos (\beta_{\nu} y_1) + B_m (\beta_{\nu} y_1) \sin (\alpha_{\nu} y_1); \quad (2.10)
\]
\[
F_2(n, y_2) = C_n \cos (\alpha_{\nu} y_2) + D_n (\alpha_{\nu} y_2) \sin (\alpha_{\nu} y_2).
\]

The coefficients \( A_0, B_0, A_m, B_m, C_n, D_n \) must be determined from the boundary conditions (2.1)-(2.5).

The functions \( u_1(y_1, y_2) \) and \( u_2(y_1, y_2) \), which specify the displacements in the stressed half-planes, can be found from a system of differential equations in the linearized theory.