The problem of the diffraction of waves by a single obstacle is difficult to solve even in an infinite region [3], and in multiply connected regions the difficulties are immeasurably increased [5]. In a semi-infinite region the solution can be constructed by using the method of images if the solution of the problem in an infinite region is known. This method has been used to solve steady-state diffraction problems [6, 7]. We note that the method of images has been applied mainly to electrodynamics problems involving point charges [4]. At the present time problems of the diffraction and scattering of waves by obstacles have been solved only in infinite regions [1, 8].

1. Statement of the Problem in a Semibounded Region. We consider fields corresponding to linear acoustic, electromagnetic, or elastic vibrations which can be described by certain scalar functions. Examples are the velocity potential in acoustics, the electromagnetic potentials, and the scalar and vector potentials in elasticity theory. In this case the diffraction problem reduces to the Cauchy problem or to a boundary value problem in a region Q for a wave equation (or system) whose coefficients have discontinuities of the first kind at some surface R.

We consider an absolutely rigid obstacle bounded by a smooth closed surface $R_1$ on which one coordinate has a constant value. The obstacle is placed in a semi-infinite region $Q_2$ with a plane boundary $R_2$. The surface $R_2$ is described in an $xyz$ Cartesian coordinate system by the equation $y = h$. The surface $R_1$ is described by a curvilinear orthogonal coordinate system with its origin coincident with the origin of the $xyz$ system. A plane wave with the fronts parallel to $R_2$ propagates from infinity along the $y$ axis. The nonstationary problem is reduced to solving the equation

$$\left( V^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \varphi = 0. \tag{1.1}$$

The function $\varphi$ can be written as the sum of two functions corresponding to the incident and scattered fields, $\varphi = \varphi_0 + \varphi_s$. We assume that at $t = 0$ the leading edge of the wave is in contact with the surface of the obstacle. Then for a plane wave we obtain

$$f(T) = \begin{cases} 0 & \text{for } T \leq 0; \\ f(T) & \text{for } T > 0, \end{cases} \tag{1.2}$$

where $T = (1/c)(y - h_1)$, and $h_1$ is the distance from the boundary $R_2$ to the outermost point of the obstacle; the function $f(T)$ in general has discontinuities of the first kind.

Thus the initial conditions take the form

$$\varphi|_{t=0} = \frac{\partial \varphi}{\partial t}|_{t=0} = 0. \tag{1.3}$$

For acoustics and electrodynamics the boundary conditions are of the Dirichlet, Neumann, or Leontovich type [2]

$$\varphi|_{R_i} = 0; \quad \frac{\partial \varphi}{\partial n}|_{R_i} = 0; \quad \left( \frac{\partial \varphi}{\partial n} + \alpha \frac{\partial \varphi}{\partial t} \right)|_{R_i} = 0 \quad (i = 1, 2). \tag{1.4}$$

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which are inhomogeneous for \( \varphi_g \) since \( \varphi \) contains the term \( \varphi_0 \). In elasticity theory Eq. (1.1) is replaced by two equations of the same type, and the boundary conditions are formulated in accordance with the first, second, and mixed boundary-value problems [1]. In addition the Sommerfeld radiation conditions must be satisfied in all the problems mentioned.


In Laplace transform space

\[
\Phi = \int_0^\infty e^{-st} q dt
\]

(2.1)

taking account of the initial conditions (1.3) the original equations take the following form:

\[
\left( \nabla^2 - \frac{s^2}{c^2} \right) \Phi = 0;
\]

(2.2)

\[
F \rightarrow f; \quad \Phi |_{\eta_i} = 0; \quad \frac{\partial \Phi}{\partial n} |_{\eta_i} = 0; \quad \left( \frac{\partial \Phi}{\partial n} + \alpha \Phi \right) |_{\eta_i} = 0;
\]

(2.3)

\[
\frac{\partial \Phi}{\partial r} + \frac{i}{c} \frac{\Phi}{r} = 0 \left( \frac{1}{V^2 r} \right) \text{ or } 0 \left( \frac{1}{r} \right) \text{ for } r \rightarrow \infty.
\]

(2.4)

We assume that the solution of problem (2.2)-(2.4) is known for an infinite region with \( i = 1 \) in (2.3). The solution of each of the three problems (2.3) for an obstacle of complex shape is very difficult. Each of the boundary conditions (2.3) may also hold on a plane surface \( (i = 2) \). We construct the solutions of these problems.

We write the solution as a convergent series

\[
\Phi = \sum_{k=1}^{\infty} \Phi_{(k)}
\]

(2.5)

where \( \Phi_{(k)} \) is the diffracted field of multiplicity \( k \).

For the Dirichlet problem we have

\[
\Phi_{(k)} (M, M*, s) = \Phi_k (M, s) + \Phi^*_{k} (M*, s).
\]

Here \( \Phi_k \) is the known solution in an infinite region, \( M \) is the point with coordinates \( \xi_i (i = 1, 2, 3) \), and \( \Phi^*_{k} \) is the image solution in the coordinate system obtained by symmetrical reflection in the plane surface.

In the Neumann problem it is convenient to go over from a scalar to a vector field. For example in an acoustic problem the potential \( \varphi \) leads to the velocity field through the equation \( \nabla \varphi = \text{grad } \varphi \). In this case we obtain

\[
\vec{V}_{(k)} (M, M*, s) = \vec{V}_k (M, s) + \vec{V}^*_{k} (M*, s).
\]

(2.6)

The problem of elasticity theory requires separate consideration. We limit our discussion to a homogeneous isotropic medium. The problem differs from (2.2)-(2.4) in that instead of (2.2) it is necessary to solve two equations of the form

\[
\left( \nabla^2 - \frac{s^2}{c^2} \right) \Phi = 0; \quad \left( \nabla^2 - \frac{s^2}{c^2} \right) \Psi = 0.
\]

(2.7)

The displacement field is determined by the expression \( \vec{U} = \text{grad } \Phi + \text{rot } \vec{B} \Psi \). The following [1] are introduced instead of (2.3):

\[
\vec{U} |_{\eta_i} = 0
\]

(2.8)

\[
U_n |_{\eta_i} = 0
\]

(2.9)

\[
T_{nk} |_{\eta_i} = 0;
\]

(2.10)

\[
U_T |_{\eta_i} = 0.
\]

(2.11)

Here \( \vec{B} \) is a linear operator, \( \vec{U} \) and \( T_{nk} \) are the Laplace transforms of the displacement vector and the components of the stress tensor, and \( U_n, U_T \) are the displacements normal and tangential to the surface.