As is known, in the case of elongation of plane specimens with a central through crack before the onset of the critical state of equilibrium (when the crack starts rapidly, and propagates in the manner of an avalanche due to a constant external load) a stage of slow steady subcritical growth of the crack is almost always observed [1]. This slow development of the crack, which is well-known to experimentors is of such an extent that the critical length of the crack \( l_c \) in relation to the properties of the material, and the length of the initial crack [5] exceeds the initial length \( l_0 \) by 30, 50, ..., 100%.

The authors of [3], [6], and [7] attach great importance to the problems of evaluating the facility of the material to inhibit fracture, that is, of evaluating the state of slow growth. At the same time this stage of the crack growth does not follow according to the existing theories of A. Griffiths, J. Irvin, M. Ya. Leonov, V. V. Panasyuk, P. M. Vitvitskii, and D. Dugdale.

In this article we have shown that it is possible to obtain a fracture diagram (the stage of slow development of a crack) from the \( \delta_c \)-theory [4].

According to the \( \delta_c \)-model, with increase of the load the initial crack \( y = 0, \ [x]_< 10 \) is detected at an invariable length to the end of the elastic displacements \( 2v(10) = \delta \) at the start of the plastic zone (where \( x = l_0 \)) reaches the limiting magnitude \( \delta = \delta_c \), which indicates that the limiting (critical) state has been reached. Only after this is it possible for the crack to propagate. It is accepted that the curve shown in Fig. 1a serves as one of the characteristics of the material. Hence the subcritical growth of the crack is not included in this diagram.

We now assume that detecting of the crack takes place simultaneously with growth of the crack, that is, the curve given in Fig. 1b will be a characteristic of the material. The form of the relationship \( \delta = \delta(l) \) in the condition \( \delta(l_0) = 0; \ \delta(l_c) = \delta_c; \ 0 \leq \delta \leq \delta_c \) must be determined experimentally. In the absence of experimental data this relationship must be approximated on the basis of certain general considerations and indirect experimental results.

To illustrate the proposed method we will assume that \( \delta(l) \) represents a section of a parabola

\[
\delta = \delta_c \left[ 1 - \left( \frac{\zeta_c - \zeta}{\beta_c - \beta_0} \right)^2 \right],
\]

where \( \zeta = l/\delta_c, \ \beta = \pi E\delta_c/(1 - \nu^2)\sigma_0, \ \sigma_0 \) is the yield point, \( E \) and \( \nu \) are the modulus of elasticity, and the Poisson ratio respectively.

Condition (1) satisfies the requirement \( \partial \delta / \partial \zeta = 0 \) where \( \zeta = \zeta_c \), which results from the physical sense of the problem.

Taking into account the results of [2] the magnitude \( \zeta_c \), which forms part of equation (1) will be determined as follows:

\[
\zeta_c = \zeta_0 (2 - e^{-5\lambda}).
\]

This equation can be argued to a certain extent by the following. As is known, in the limit when \( l_0 \rightarrow \infty \) the
theory of A. Griffiths with a constant density of the fracture energy follows from the \( \delta_c \)-theory. Examining the subcritical state in which each \( p = p(l) \) corresponds with the steady state of equilibrium, we can write (by somewhat extending the formulation of the Griffiths equation)

\[
\left( \frac{\partial}{\partial l} + \frac{\partial}{\partial p} \frac{dp}{dl} \right) \left( 2vp - \frac{\pi E p^2}{2} \right) = 0.
\]

Hence with the condition that \( p = 0 \) when \( l = l_0 \) we have the analytical expression for the fracture diagram \( p = 4\alpha_0\sqrt{c(l - l_0)/\pi l} \). As we see, at any initial length \( l_0 \) the development of the crack to a final length \( l_c \) is 100%.

The equations of the fracture diagram in accordance with the \( \delta_c \)-theory, and the generalizations adopted here are obtained from the condition

\[
2v(l) = \delta. \tag{3}
\]

We will dwell on some individual cases.

**Case 1.** An infinite plane surface with an isolated rectilinear crack is subjected to the influence of a stress \( p \) perpendicular to the line of the crack. Using the value \( v(l) \) known from [4], based on the equations (1) and (3) we obtain

\[
\lambda = \frac{2}{\pi} \arccos e^{-\Delta}, \tag{4}
\]

where

\[
\lambda = \frac{E}{\alpha_0}, \quad \Delta = \frac{1}{2} \left[ 1 - \left( \frac{\varepsilon_0 - \varepsilon_c}{\varepsilon_0 - \varepsilon_c} \right)^2 \right].
\]

The fracture diagrams plotted for various values \( \varepsilon_c \) according to formula (4) are shown in Fig. 2 (curves 1). The maxima of these diagrams are located on the line of the limiting stresses [4] (curve 2).

**Case 2.** An infinite plane surface with a rectilinear crack, in whose plane acts an internal pressure \( p \). The movement \( v(l) \) is determined by known methods. Hence equation (3) has the form

\[
-(1 + \lambda) \ln \cos \frac{\pi \lambda}{2(1 + \lambda)} = \Delta. \tag{5}
\]

The fracture diagrams for this case are given in Fig. 2 by curves 3, and the limiting loads by curve 4.

**Case 3.** An infinite space with a disc-shaped (circular in plan view) crack, situated in the field of influence of tensile stresses \( p \) in a direction perpendicular to the plane of the crack. In such a case, using the results of [4] and [9] based on the equations (1) and (3) we will find

\[
1 - V \sqrt{1 - \lambda^2} = \Delta. \tag{6}
\]