Fluid and viscoelastic dampers employ silicones, since the viscosities of these fluids are relatively temperature-independent. Silicone viscosity does depend on the velocity gradient, however.

The problem of selecting optimal parameters for a viscoelastic torsional-vibration damper is solved for a system with three degrees of freedom on the assumption that the fluid viscosity is nonlinear. With the formulas obtained it is possible to determine the optimal viscosity and stiffness, and thus to determine the amplitude of torsional vibrations. It is shown that a viscoelastic damper with nonlinear viscosity is more effective than a fluid damper.

Figure 1 shows a torsional-vibration damper employing friction and elastic coupling. The flywheel mass 4 of the damper (a ring) is seated freely on hub 3 and is rotated by the moment due to the liquid-friction forces and to the elastic forces 2. In the presence of torsional vibrations, the mass tends to rotate at constant speed; the hub and case 1, which are rigidly connected to the shaft, also execute oscillatory motion in addition to uniform rotation. The oscillatory motion of the damper ring with respect to the hub and the case, which produces fluid-friction and elastic forces, dissipates the mechanical energy of the shaft torsional vibrations. Figure 2 shows an arrangement of two masses and a viscoelastic damper. The crankshaft system of a radial motor reduces to such an arrangement [1].

As has been shown [8], the viscosity of a silicone depends upon the velocity gradient. Thus, for example, as the velocity gradient ranges between 20 and 10 000 sec⁻¹, the viscosity of one grade of silicone (at the same temperature) varies by roughly a factor of 6.

We assume that the viscous moment in the damper depends on the relative speed of the ring and hub (the possibility of representing the viscous moment in this way has been shown elsewhere [3, 8]):

\[ M_v = c(\dot{\varphi}_1 - \dot{\varphi}_3) - \varepsilon(\dot{\varphi}_1 - \dot{\varphi}_3)^3, \]

where \( \varphi_1 \) and \( \varphi_2 \) are the angular deviations of the damper case and ring; \( c \) and \( \varepsilon \) are coefficients.

Then the equations of motion for the two masses and the damper ring take the form

\[
\begin{align*}
I_1 \ddot{\varphi}_1 + k_1 (\varphi_1 - \varphi_3) + k_2 (\varphi_1 - \varphi_4) + \\
+ c(\dot{\varphi}_1 - \dot{\varphi}_3) - \varepsilon(\dot{\varphi}_1 - \dot{\varphi}_3)^3 &= P \cos \omega t, \\
l_1 \ddot{\varphi}_2 + k_2 (\varphi_2 - \varphi_4) + c(\dot{\varphi}_2 - \dot{\varphi}_1) - \varepsilon(\dot{\varphi}_2 - \dot{\varphi}_1) &= 0, \\
l_2 \ddot{\varphi}_3 + k_1 (\varphi_3 - \varphi_2) &= 0.
\end{align*}
\]

Here \( I_1, I_2, \) and \( I_3 \) are the moments of inertia of the first mass, the damper ring, and the second mass; \( k_1 \) is the stiffness of the shaft; \( k_2 \) is the stiffness of the damper elastic link; \( \varphi_3 \) is the angular deviation of the second mass; \( P \cos \omega t \) is the harmonic disturbing moment.

We let \( \varphi_1 - \varphi_2 = \alpha_1 \) and \( \varphi_1 - \varphi_3 = \alpha_2 \). In the new coordinates, the system (1) will take the form

\[
\begin{align*}
l_1 I_2 \ddot{\alpha}_1 + l_1 I_2 \ddot{\alpha}_2 + k_2 (l_1 + l_2) \alpha_1 + \\
+ c(l_1 + l_2) \dot{\alpha}_2 - \varepsilon(l_1 + l_2) \alpha_2^3 &= P l_1 \cos \omega t, \\
l_2 \ddot{\alpha}_2 - k_2 l_2 \alpha_2 - cl_2 \dot{\alpha}_2 + k_1 l_2 \alpha_2 + \varepsilon l_2 \dot{\alpha}_2^3 &= 0.
\end{align*}
\]

We go over to dimensionless variables in the last two equations, introducing the dimensionless time \( \tau = \omega t \).

They then look like this:

\[
\begin{align*}
l_1 \omega^2 \ddot{\alpha}_1 + k_1 l_2 \alpha_1 + k_2 (l_1 + l_2) \alpha_1 + c(l_1 + l_2) \dot{\alpha}_1 - \\
- \varepsilon \omega^2 (l_1 + l_2) \alpha_1^3 &= P l_1 \cos \tau, \\
l_2 \omega^2 (\alpha_2 - \alpha_2') - k_2 l_2 \alpha_2 - c l_2 \dot{\alpha}_2' + k_1 l_2 \alpha_2 - \varepsilon l_2 \dot{\alpha}_2'^3 &= 0.
\end{align*}
\]
where the primes indicate derivatives with respect to $\tau$.

Multiplying the first equation of (2) by $k_1 I_1$, and the second by $(I_1 + I_2) I_1^{-1} k_1^{-1}$, we find that

$$y^2 a_d' + x + \beta a_d = \lambda_1 \cos \tau,$$

$$y^3 b_d' + 2 \mu y a_d' = \frac{\lambda_1}{y^3} \cos \tau,$$

$$\beta y^3 (a_d' - a_d') - \beta a_d' = 2 \mu y a_d' + \beta y a_d + \epsilon a_d^3 = 0,$$  \hspace{1cm} (3)

where $\gamma = \omega (I_1 / k_1)^{1/2}$ is the ratio of the disturbing-force frequency to the natural frequency of the first mass; $\delta = (k_2 I_1 / k_1 I_2)^{1/2}$ is the ratio of the natural frequencies of the ring and the first mass; $\beta = (I_1 + I_2) / I_1$ is the ratio of the sum of the moments of inertia of the first mass and ring to the moment of inertia of the first mass; $\mu = (c / 2 I_2) (I_1 / k_1)^{1/2}$ is the viscous resistance; $\beta_1 = I_1 / I_3$ is the ratio of the moments of inertia for the first and second masses; $\lambda_{st} = P / k_1$ is the static deviation of the first mass resulting from force $P$.

We linearize system (3), using the den Hartog method [2]. We assume that both masses and the damper ring move harmonically:

$$\alpha = X_0 \cos (\tau + \varphi_0), \quad a_d = Y_0 \cos (\tau + \varphi_0 + \varphi_0).$$

We replace the damper resistance moment $\beta a_d' + 2 \mu y a_d' - \epsilon a_d^3$ by the equivalent moment $\beta a_d' - \epsilon a_d^3$ (i.e., with the constant coefficient $c_1$), so that for harmonic motion, these moments do equal work:

$$\int_0^{2\pi} \epsilon a_d^3 \, da = \int_0^{2\pi} c_1 a_d' \, da.$$  \hspace{1cm} (6)

From this we obtain

$$c_1 = \frac{3}{4} \epsilon V^2 \gamma^3.$$

After such linearization, (3) takes the form

$$y^2 a_d' + x + \beta a_d = \lambda_1 \cos \tau,$$

$$y^3 b_d' - (2 \mu y - c_1) a_d' = \lambda_1 \cos \tau,$$

$$\beta y^3 (a_d' - a_d') - \beta a_d' = (2 \mu y - c_1) a_d' + \beta y a_d = 0.$$  \hspace{1cm} (4)

We write the forced vibrations of (4) as

$$\alpha = X_0 e^{\omega t}; \quad a_d = Y_0 e^{\omega t}.$$  \hspace{1cm} (5)

Substituting these coefficients into (4) and equating the coefficients, we find that

$$\begin{bmatrix} X_0 & Y_0 \end{bmatrix} \begin{bmatrix} \beta (\gamma - \gamma_0) + i (2 \mu y - c_1) \lambda_1 \end{bmatrix} + \lambda_1 = \lambda_1.$$  \hspace{1cm} (6)

The solution of (5) has the form

$$X_0 = \begin{bmatrix} \beta (\gamma - \gamma_0) + i (2 \mu y - c_1) \lambda_1 \end{bmatrix} \times$$

$$\times \left\{ \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2) +$$

$$+ i \beta (\gamma - \gamma_0) (2 \mu y - c_1) \right\},$$

$$Y_0 = \begin{bmatrix} \beta (\gamma - \gamma_0) \end{bmatrix} \beta (\gamma - \gamma_0) \left( \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) +$$

$$+ \beta (\gamma - \gamma_0) (2 \mu y - c_1) \right).$$  \hspace{1cm} (7)

Consequently, the effective amplitudes are found from the formulas

$$X_0 = \lambda_1 \left\{ \beta (\gamma - \gamma_0) + \beta (\gamma - \gamma_0) \left[ \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) +$$

$$+ \beta (\gamma - \gamma_0) (2 \mu y - c_1) \right] \right\}.$$  \hspace{1cm} (8)

The amplitude $X_0$ is independent of $\mu$.

Equation (8) breaks down into two equations

$$\left\{ \begin{array}{l}
\beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\gamma - \gamma_0) (2 \mu y - c_1) = 0, \\
\beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\gamma - \gamma_0) (2 \mu y - c_1) = 0.
\end{array} \right.$$  \hspace{1cm} (9)

We note that if $\gamma \to 0$, then $X_0 \to 0$, i.e., when the free-vibration frequency of the first mass is far from the disturbing-force frequency, there is no need to damp torsional oscillations. When $\mu = c_1 = 0$, from (6) we have

$$X_0 = \frac{\lambda_1 (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2)}{(\beta_1 - \gamma_0) (\delta^2 - \gamma^2) + (\delta^2 - \gamma^2) \gamma^2}.$$  \hspace{1cm} (10)

As we see from (7), $X_0 \to 0$ as $\gamma \to \delta$. If $\gamma$ approaches one of the roots of the equation $(\beta_1 - \gamma_0) (\delta^2 - \gamma^2) + (\delta^2 - \gamma^2) = 0$, however, then $X_0 \to \infty$, i.e., in this case the viscoelastic damper becomes a linear vibration mounting [4], which is effective only within a narrow frequency band.

In accordance with (6), we find the partial derivative

$$\frac{\partial X_0}{\partial \mu} = 2 \lambda_1 \left\{ \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2) \gamma^2 +$$

$$+ \beta (\gamma - \gamma_0) (2 \mu y - c_1) \right\}.$$  \hspace{1cm} (11)

where

$$B = \beta (\beta_1 - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2).$$

Thus we have that when $\gamma$ satisfies

$$|\beta_1 - \gamma_0| \left( \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2) \gamma^2 =$$

$$= (\gamma^2 - \delta^2) (\beta_1 - \gamma_0) (\delta^2 - \gamma^2),$$  \hspace{1cm} (12)

the amplitude $X_0$ is independent of $\mu$.

Equation (12) breaks down into two equations

$$\begin{bmatrix} \beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2) \gamma^2 = 0, \\
\beta (\gamma - \gamma_0) (\delta^2 - \gamma^2) + \beta (\delta^2 - \gamma^2) = 0.\end{bmatrix}$$  \hspace{1cm} (13)

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