NONLINEAR BEHAVIOR OF A SUPERCONDUCTOR
IN AN ELECTROMAGNETIC FIELD

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It will be shown in this paper that it is possible to use a simpler equation than the Gor'kov-Bogolyubov equation for the description of the behavior of a superconductor in an electromagnetic field in the BKSh model far from the critical temperature $T_c$. An "exact" solution of this equation has been found for fields that vary in space. The possibility of using perturbation theory for the solution of this problem is analyzed. It turns out that this possibility is generally absent at frequencies $\omega$ lying in the shf region and lower, even in the case of comparatively low powers. The problem of the nonlinear behavior of a superconductor in an electromagnetic field is of appreciable theoretical and practical interest. However, as far as the author knows, only individual mechanisms [1, 2] that lead to appreciable nonlinearities have been discussed up to now. In contrast to the indicated papers, a scheme is developed in this paper that permits considering this problem for a London superconductor in the general form. A conclusion is drawn about the conditions necessary for the solution of this problem according to perturbation theory [3] on the basis of an analysis of the solution of the equation derived.

GENERAL FORMALISM

It is convenient for the description of the behavior of a superconductor in an electric field to introduce [4] a correlation function in the Nambu representation for coincident instants of time in the form

$$\rho(x, y, t) = \langle \hat{\Phi}^+(x,t) \Phi(y) \rangle - \frac{1}{2} \delta(x-y).$$

(1)

and $\hat{\Phi}(x) = \hat{\Phi}(x)$ is the particle annihilation operator at the point $x$ in the Nambu representation.

Knowledge of the correlation function permits us to find all physical quantities of a single-particle nature. Thus, we have for the current density

$$j(x) = \frac{ie}{m} \nabla_x \rho(x, y, t)|_{x=y} = \frac{eN}{mc} A(x, t).$$

(2)

Let us introduce the matrix $U(x, p)$, which describes the evolution of the operator according to the equation

$$\hat{\Phi}(x) = \int U(x, p) \gamma(p) \frac{d^3 p}{(2\pi)^3},$$

(3)

where

$$\gamma(p) \equiv \begin{pmatrix} b_p & 1 \\ b_p^* & 0 \end{pmatrix}.$$

The symbol $\langle \ldots \rangle$ in Eq. (1) and in the following denotes an averaging operation with respect to the density matrix.
The eigenoperators for the instant \( t = -\infty \) are \( b_{p^+} \), i.e.,

\[
\langle \gamma^+ (p) \gamma (p') \rangle = \left[ \mu (p) z_2 + \frac{1}{2} (1 - z_2) \right] \delta_{pp'}.
\] (4)

As follows from Eqs. (1), (4), and (3), \( U(x, p) \) is the evolution matrix for the single-particle density matrix in the mixed coordinate--momentum representation. It is possible to write for it the Gor'kov--Bogolyubov equation, which results from the equation of motion for the Heisenberg operators \( \hat{\phi} (x) \). These equations can be represented in the form

\[
i \left. \frac{\partial}{\partial t} U(x, p) \right|_{t=0} = \frac{eA(x)}{mc} \frac{\hbar}{\hbar} + \frac{e\Phi(x)}{mc} \frac{\hbar}{\hbar} + \Delta_0 z_2 + \Delta (x) z_+ + \Delta^* (x) z_- U(x, p).
\] (5)

Here \( \sigma_z \) and \( \sigma_x \) are the Pauli matrices, \( z_\pm = \frac{1}{2} (z_x \pm i z_y) \); \( \Delta_0 \), and \( \Delta (x) \) are the initial value of the order parameter and its variation upon application of the electromagnetic field, respectively, and \( \mu \), \( \Lambda (x) \), and \( \Phi (x) \) are the chemical, vector, and scalar potentials, respectively.

Equation (5) should be satisfied by the self-consistency equation

\[
\Delta_0 + \Delta (x) = \lambda \text{Sp} z_+ \rho (x, y, t) |_{x=y},
\] (6)

and also by Maxwell's equations for an electromagnetic field.

Substituting (3) into (1) with (4) taken into account, we have

\[
\rho (x, y, t) = - \frac{1}{2} \int U^* (x, p, t) z_2 \frac{\omega_p}{\omega_T} \frac{d^3p}{d^3x} U(y, p, t) \frac{d^3p}{(2\pi)^3}.
\] (7)

in the case of a superconductor. Here \( \omega_p = \sqrt{\frac{\hbar^2}{m^2} + \Delta_0^2} \); \( \xi (p) = \omega_p (p - p_0) \), and \( T \) is the temperature. The bar denotes transposition of the matrix indices.

We will seek the solution of Eq. (5) in the form

\[
U(x, p) = U_0 (x, t_0) \left[ \delta (x - p) + T (x, p, t) \right] U_c (x, p).
\] (8)

where \( T(x, p, t) \) is the transition matrix and \( U_0 (x, p) \) is the solution of the equation

\[
i \left. \frac{\partial}{\partial t} U_0 (x, p) \right|_{t=0} = \frac{eA(x)}{mc} \frac{\hbar}{\hbar} + \frac{e\Phi(x)}{mc} \frac{\hbar}{\hbar} + \Delta_0 \sigma_x U_0 (x, p).
\] (9)

One can write \( U_0 (x, p) \) in the form

\[
U_0 (x, p) = U(p) \exp \left( p r - \omega_p \sigma_z t \right).
\] (10)

Here \( U(p) \) is the Bogolyubov transformation matrix,

\[
U(p) = \left( \begin{array}{c} u_{pp'} \\ \nu_{pp'} \end{array} \right), \quad u_p = \frac{1}{\sqrt{2}} \left( 1 + \frac{\xi (p)}{\omega_p} \right)^{1/2}, \quad \nu_p = \frac{1}{\sqrt{2}} \left( 1 - \frac{\xi (p)}{\omega_p} \right)^{1/2}.
\] (11)

Substitution of (8) into (5) with (9) and the unitarity of \( U_0 (x, p) \) taken into account gives

\[
T(p, p', t) = T^{(c)} (p, p', t) + \int_{-\infty}^{t} \int T^{(c)} (p, \kappa, t') T (\kappa, p', t') \frac{d^3\kappa}{(2\pi)^3} dt',
\] (12)

where

\[
T^{(c)} (p, p', t) = -i \int U_0 (x, p) \hat{V} (x) U_0 (x, p') dV.
\] (13)