CONTINUED FRACTIONS IN THE KINETIC THEORY OF
SUPERSONIC AMPLIFICATION IN ANISOTROPIC
SEMICONDUCTORS

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The method previously developed by the author (Fiz. Tekh. Poluprovodn., 9, 360, 1975; 10, 808, 1976) is applied to evaluate the electron coefficient of supersonic absorption (or of amplification) with arbitrary $q_l$ ($Q_e$) and of acoustic electric current ($j_{ae}$) in a single-pole "one-valley" semiconductor in the case of no external electric field $E_0$. The anisotropy of the acoustic-electronic interaction (the piezo-electrical one and the deformation potential) is taken into account as well as the anisotropy of scattering and of the effective mass. In contrast to the Zil'berman—Mishin results (Zh. Éksp. Tekh. Fiz., 65, 1473, 1973; Fiz. Tekh. Poluprovodn., 10, 1882, 1976) which were obtained for the isotropic case, compact expressions are obtained for $Q_e$ and $j_{ae}$ in the form of an infinite continuous fraction of simple structure with all the spherical harmonics and all their relaxation times ($\tau_n$) taken into account. The summation of the fraction in the Zil'berman—Mishkin approximation converts the latter into a finite continuous fraction yielding at once any order of this approximation which simplifies very considerably the analysis of its convergence. The taking into account of anisotropy does not really complicate the result and maintains its compact form. Another method of fraction summation enables one to estimate the contribution of individual harmonics.

The technical mastery of hypersonics was instrumental in developing a theory of acoustic-electronic effects within a wide range of frequencies including any values of $q/l_p$ ($q$ is the wave sound vector, $l_p$ is the impulse length of the carriers free path). Such a theory calls for higher spherical harmonics of the distribution function for current carriers ($f_{jk}$) to be appropriately taken into account; the latter was first carried out in [1, 2]. There is a tendency to smooth out various relaxation times of individual harmonics ($\tau_n$) with $n$ increasing. If for all $n > \nu$ (where $\nu$ is a constant integer) one sets $\tau_n = \tau = \lim_{n \to \infty} \tau_n$, then the infinite system of algebraic equations for the expansion coefficients $j_k$ is reduced to a finite system of ($\nu + 2$) equations. The convergence of this "$\nu$ approximation" for $\nu \to \infty$ has, however, not proved sufficient for all scattering mechanisms. It was subsequently shown in [3] that this inadequacy can be obviated by the modified $\nu$ approximation:

$$\tau_n = \tau_{\nu+1} \quad (\text{at } n > \nu).$$

Nevertheless, the method in [1-3] has, on the whole, the following disadvantages: 1) For each value of $\nu$ a large system of ($\nu + 2$) equations must be formed and solved. With $\nu$ increasing the bulk of the computations and of the results grows rapidly. This hampers the application of the results as well as the convergence analysis of the $\nu$ approximation; 2) the method given in [1, 2] was developed only for the isotropic case which narrows considerably its region of applicability; 3) the method does not enable one to estimate the contributions of individual harmonics (for given intervals $q/l_p$). All these disadvantages disappear in the continued-fractions method as developed in [4, 5]. One is then able to obtain compact and elegant expressions for acoustic-electronic effects by means of infinite continued fractions of a very simple structure. Such fractions take into account all the higher harmonics and all $\tau_n$ and does it exactly without the $\nu$ approximation. If the continued fraction is summed by using the $\nu$ approximation, as proposed in [5], the fraction is converted into a finite continued fraction with $\nu$ units. If $\nu$ increases by one, then one unit is added and the result is obtained "almost instantaneously." There is no need to build up and to solve another bulky system of ($\nu + 2$) equations. Moreover, the structure itself of a continued fraction admits an additional approximation which enables one to simplify the obtained result; the latter becomes more effective the greater $\nu$!

It is thus much easier to analyze the convergence of an $\nu$ approximation and to apply the obtained results. However, if the continued fraction is summed by successive cut-offs [4, 5] the possibility arises of estimating the contributions of individual harmonics.

Finally, the method of [4, 5] can easily be generalized to anisotropic cases (including radiative anisotropy) which extends rapidly the applicability region. It is interesting to note that by using such generalizations the result is hardly more complicated fully maintaining its general form. Thus, the obtained result with a much higher accuracy than in [1, 2] is incomparably simpler.

In [4, 5] the continued-fractions method was applied to evaluate absorption (and dispersion) of USW in the isotropic case with no external fields. In the present article the method is applied to evaluate absorption (or amplification) of the ultrasonic waves and to compute acoustic-electric current in an anisotropic crystal with the external electric field $E_0$ present. As in [1-5] the computation is carried out for the region in which the energy relaxation, the electron collision, and the effect of traps can be neglected.

By solving the system of equations which consists of 1) the motion of an elastic piezoelectric medium with a "deformation-potential" force present, 2) the Poisson equation, and 3) the kinetic equation, the electron coefficient of absorption $\Gamma_\nu$ is expressed by means of the linking constants $C_p$ and $C_d$, the stationary distribution function of the carriers in the field $E_0(\mathbf{r})$, and the resolvent $\hat{R}$ of the "kinetic operator" $\hat{L}$:

$$\Gamma_\nu = \Gamma_0 \Delta_0^{-1} S; \quad \Gamma_0 = \frac{qn_x(C_x + C_d)}{\rho v_0 k_0 \tau};$$

$$\Delta_0 = [1 + (qr_0)^{-2}S]^{-1} + (qr_0)^{-4}S; \quad n_x = \frac{\partial n_0}{\partial z};$$

$$S = S_r + iS_i; \quad \hat{R}^D = 1; \quad r_{D}^{-2} = \frac{n_x e^2}{\chi_{eq} k_0 \tau};$$

$$S = \left( 1 - \frac{ik_0 \tau}{4\pi^2 n_x} \int \hat{R}(q\nu_p) f_0 d\nu \right); \quad \hat{L} = \hat{L}_0 + \hat{L}_1;$$

$$\hat{L}_0 = \hat{I} + i(q\nu - \omega); \quad \hat{L}_1 = eE_0\nu_p;$$

$$C_p = \frac{\partial \chi_{eq}}{\chi_{eq} k_0 \tau}; \quad C_d = C_{eq}.$$

In the above $\hat{I}$ is the collision operator; $v$ is the carrier velocity; $\rho$ is the crystal density; $v_0$ is the sound velocity; $n_0$ is the equilibrium concentration of the current carriers; $\beta_{ij}$ (piezoelectric), $\chi_{ij}$ (dielectric constant), $C_{ij}$ (the deformation-potential constant) with the vector $e_0$ of sound polarization and the vector $q^0 = qq^{-1}$; $z$ is the reduced chemical potential. The result (1) holds for any degree of Fermi carrier degeneration.

Thus, the problem has been reduced essentially to the evaluation of $\hat{R}$. By expanding $\hat{R}$ in the powers $E_0$ one can express it by means of the resolvent $\hat{R}_0$ of the fieldless problem:

$$\hat{R} = \sum_{n=0}^{\infty} \hat{R}_n; \quad \hat{R}_n = - \hat{R}_0 \hat{L}_1 \hat{R}_{n-1} \ldots = (- \hat{R}_0 \hat{L}_1)^n \hat{R}_0; \quad \hat{R}_0 \hat{L}_0 = 1.$$

Having expressed the odd part of $\hat{f}_{k}$ by an even one which is replaced by the isotropic harmonic $f_0(\varepsilon)$ as in [1, 2] one linearizes $S$ with respect to $E_0$ which does not appear in $f_0(\varepsilon)$. To evaluate $\hat{R}_0$ one employs the following expansion:

$$\hat{R}_0 Y_{lm}(\theta, \varphi) = \sum_{l', m'} R_{l'l'}^{m'm} Y_{l'm'}^{*}(\theta, \varphi);$$

(and similarly for $\hat{L}_0$), where $\theta, \varphi$ are the spherical coordinates of the quasimomentum $k$. Then the following system is obtained for $R_{l'l'}^{m'm}$:

$$\sum_{l', m'} R_{l'l'}^{m'm} L_{l'l'}^{m'm'} = \delta_{l', l'} \delta_{m', m}.$$

One can easily find $L_{l'l'}^{m'm'}$ explicitly by expanding the scatter probability $w_{kk'}(\varepsilon - \varepsilon')$ (see [6]):

$$w_{mm'} = \sum_{l, l'} W_{l'l'}^{m'm'} Y_{lm}(\theta, \varphi) Y_{l'm'}^{*}(\theta', \varphi').$$