The summing of the infinite fraction $P_n^0$ by using the $\nu$ approximation and the replacement $r_s \rightarrow 1$ ($s \geq 2$) yields for any $\nu$ [5]:

$$P_n^0 \approx a_s^2 + [a_s^2 + (a_s^2 + \cdots + (a_s^{(0)} + \bar{P}_s)^{-1} \cdots)^{-1}]^{-1};$$

$$\bar{P}_s \equiv a^{-1}[1 + (1 + u^2)]^{-1}; \quad u \equiv q v v_{\nu+1}(x); \quad a_s^2 = [a_s]_{\nu-\infty}^2. \quad (9)$$

Formula (9) is a finite continued fraction which can produce at once any order of the $\nu$ approximation; the latter simplifies greatly the convergence analysis with $\nu \rightarrow \infty$.

In the isotropic case for $\nu = 0, 1, 2$ one can obtain the results of [1-3] from (7)-(9), respectively (with deviations not exceeding a few percent for any $q/p$). For $E_0 = 0$ the results of [4, 5] are obtained. In the anisotropic case for $q_{dp} < 1$, when the contribution of higher harmonics can be neglected one finds from (7)-(9) the results of [7, 8] (for $B = 0$ and the corresponding orientation of $q$).

The estimates of the contributions of individual harmonics by using the method of [5] produce more or less the same results as for $E_0 = 0$.

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LITERATURE CITED


DISTRIBUTION FUNCTIONS AND GREEN'S FUNCTIONS

B. A. Veklenko

A unitary transformation is indicated which makes it possible when computing a partially traced density matrix to use the method of two-time quantum Green's functions. For systems in strong nonequilibrium, an integral equation is found which describes both the initial and kinetic stages of relaxation of the density matrix of a subsystem from any initial distribution.

The method of two-time Green's functions is the most powerful tool for investigating averaged bilinear products of field operators [1-3]. If, however, the problem requires invoking correlators of higher orders, then this method actually ceases to serve, since its simplest generalization already leads to a Bethe-Salpeter equation which is hard to solve. In such problems (quantum generators in the threshold region, cooperative effects, etc.) the equation for the density matrix is investigated directly but the generality and beauty of the methods inherent in the two-time Green's functions are hereby lost.

In [4] a unitary transformation was indicated which makes it possible to compute a partially traced density matrix using the machinery of two-time Green's functions. All its attractive features such as Feynman diagrams, spectral theorems, summation rules, the Dyson equation, etc. are hereby preserved. To illustrate the method in [5] a derivation of a linear version of the Scully-Lamby equation [6] was given, and the corresponding diagram technique was studied.

A complete theory is present below which makes it possible under conditions arbitrarily far from equilibrium to invoke a method analogous to the method of the kinetic Green's functions in a form suggested by Kaldysh [7] to compute the density matrix.

An integral equation is obtained which describes the evolution of a partially traced density matrix from an arbitrary initial state.

As a model, we consider a gas of alkali atoms interacting with a resonance transverse electromagnetic field. If we denote by \( \psi(X) \) and \( A_{\mu}(x) \) the corresponding Heisenberg field operators of the atoms and photons, \( X = \{r, R, t\} \), \( x = \{r, t\} \), \( r \) and \( R \) are the coordinates of a valence electron and the atomic balance, then the Hamiltonian of the system can be represented in the form \( \hbar = c = 1 \) [4]

\[
H = \int \psi^+ \left( H_0 - \frac{e}{m} P_\mu A_\mu \right) \psi \, dr \, dR + \frac{1}{2} \int \left( \frac{\partial A_\mu}{\partial r} \frac{\partial A_\nu}{\partial r} + \frac{\partial A_\mu}{\partial t} \frac{\partial A_\nu}{\partial t} \right) \, dr.
\]

Here \( H_0 \) is the energy operator of the atoms and \( P_\mu \) is the momentum operator of the electron. The type of statistics of the \( \psi \) field has no meaning for nondegenerate gases. To be specific, we assume that the atoms are bosons. We turn to the Schrödinger representation. Then

\[
\Phi(\zeta) = \sum_N B(N) \tilde{\Phi}_N(\zeta),
\]

which realizes the transition to the new representation. Here \( B(N) \) is the operator which annihilates the collection of photons in the state \( \tilde{\Phi}_N(\zeta) \) and acts in some space \( \Gamma \). Further, \( B^+(N) \psi \) is the wave function in this state corresponding to the state \( \psi^+ N(\zeta) \).

In the \( \Gamma \)-representation the average value of any second-quantized photon operator \( K \) has the form

\[
\langle K \rangle = \int \langle \Phi^+(\zeta) K(\zeta', \zeta') \Phi(\zeta') \rangle \, d\zeta'' \, d\zeta', \quad d\zeta'' = \prod_{\alpha \lambda} d\zeta''_{\alpha \lambda},
\]

where \( \langle \rangle \) denotes the wave function of the system in \( \Gamma \) space. Thus, the construction

\[
\langle \Phi^+(\zeta) \Phi(\zeta') \rangle = \rho
\]

plays the role of a density matrix traced over the atomic arguments. To compute (1) it is convenient to go over to the Heisenberg representation. Then [4]

\[
i \frac{\partial \Omega}{\partial t} = H_0 \Omega - \frac{e}{m} \int \phi^+ P_\mu A_\mu \phi \, d\zeta, \quad \Omega = \int \psi^+ \psi \, dr \, dR \phi,
\]

\[
i \frac{\partial \Phi}{\partial t} = H_{ph} \Phi - \frac{e}{m} \int \psi^+ P_\mu A_\mu \psi \, dR \Phi,
\]

\[
H_{ph} = \sum_{\alpha \lambda \kappa \nu} \left( \zeta_{\alpha \lambda \kappa} \zeta_{\nu \lambda \kappa} + \frac{1}{2} \right),
\]

\[
[\psi, \psi^+]_{r, r'} = \delta(r - r') \delta(R - R'), \quad [\Phi, \Phi^+]_{r, r'} = \prod_{\alpha \lambda \kappa \nu} \delta(\zeta_{\alpha \lambda \kappa} - \zeta_{\nu \lambda \kappa}).
\]

In \( \Gamma \) space we construct the Green's function

\[
\tilde{D}_{\mu}(\zeta, t, \zeta', t') = -i \langle T_T \phi(\zeta, t) \phi^+(\zeta', t') \rangle.
\]