The necessary and sufficient condition for compatibility of (15) and (16) is
\[ i \int \frac{[D(E+i0)-D(E-i0)]}{2\pi} \frac{dE}{e^{-\frac{E}{\Theta}+\eta}} = 1, \]
which agrees with representation (13). We finally find
\[ \rho(N) = Q^{-1} \left\{ \int \frac{D(E+i0)-D(E-i0)}{2\pi} \frac{dE}{e^{-\frac{E}{\Theta}+\eta}} \right\}^{-1} - \eta \].

The value of the statistical sum $Q$ follows from the condition
\[ \sum_N \rho(N) = 1, \]
which expresses in agreement with (3) the average value of the identity operator. The explicit dependence of $\rho(N)$ on $\eta$ was a consequence of the approximate character of the calculations related to elementary decomposition of the chain of equations. That the parameter $\eta$ drops out in the result bears witness to the inner consistency of the approximation. For example, if for a weak interaction we consider a sharp almost $\delta$-type character of the function $D(E+i0)-D(E-i0)$ at the point $E = \varepsilon(N)$, then from (14) we find
\[ \rho(N) = Q^{-1} \left( -\frac{\omega N}{\Theta} + \frac{1}{\Theta} \right) e^{\frac{\omega N}{\Theta}}. \]
Here the operator $\Pi_q$ is taken at the point $E = \omega_q N$. In this approximation the distribution of quasiparticles over the various states turns out to be mutually independent. It is possible to pose the question of the optimum choice of the parameter $\eta$, defining it, e.g., from the condition
\[ \frac{\partial Q}{\partial \eta} = 0. \]

For the computation of $N_{\rho\eta}$ a formalism arises which is similar to that expounded in [2].

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LITERATURE CITED


CHRONOGEOMETRY AND THE DYAD METHOD
IN THE THEORY OF RELATIVITY

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A connection of the dyad method and chronogeometry is revealed which is based on the definition of distance by means of an observer's measurement of the time of emission and reception of a signal, propagating along isotropic geodesics and reflected from an observed subject. A number of examples of defining distance by a moving observer are considered. It is shown that the distance and time in chronogeometry defined between nearby points of a reference frame are equivalent to distance and time in the monad method of prescribing the reference frame.

Various authors have repeatedly emphasized the dominant role of time in physical measurement. Thus, J. L. Synge observes: "For us the only basic measure is time. Length (or distance), insofar as it is necessary..."
and desirable to introduce it, will be considered as a derived concept." And further on: "We are actually dealing with Riemannian chronometry rather than geometry..." [1]. Penrose [2] and a number of other gravitationalists have expressed themselves in a similar way.

On the other hand, to describe concrete effects in the theory of relativity, it is necessary to define a reference system which is most economically done by the monad method [3]. If necessary the dyad [4] or tetrad [5] methods are used. These methods make it possible to interpret geometric quantities and results of measurements. In this connection there arises the problem of comparing the devices of chronogeometry or, what we will take to be the same, the problem of comparing chronometry with monad and dyad methods in the theory of relativity.

Chronogeometry is based on determining distance by means of measurement by an observer of the moments of time of emission and reception of a signal propagating along isotropic geodesics which is reflected by an observed object. This method of determining distance is universal in the sense that it can be used both under laboratory conditions and under conditions precluding the possibility of another method of measurement.

Suppose that the observer moves along a world line \( x^\alpha (s) \), where \( s \) is the canonical parameter of this line defining the proper time of the observer. The chronometric method of determining time and distance is related to the problem of finding the points of intersection of the light hypercone with vertex at the point \( y^\alpha \) observed with the world line of the observer \( x^\alpha (s) \). If the equation of the hypercone with vertex at the point \( y^\alpha \) has the form \( E(x^\alpha, y^\alpha) = 0 \), then the equation defining the value of the parameter \( s \) at the points of intersection has the form \( E[x^\alpha(s), y^\alpha] = 0 \).

We assume that there exist two solutions of this equation \( s_1 \) and \( s_2 \) corresponding to the time of transmission of the signal and the time of reception of the signal reflected from the object. Then the distance from the observer to the point \( y^\alpha \) we define to be

\[
R(y^\alpha) = \frac{s_2(y^\alpha) - s_1(y^\alpha)}{2}. \quad (1)
\]

The moment of proper time of the observer to which this distance corresponds we put equal to

\[
T(y^\alpha) = \frac{s_2(y^\alpha) + s_1(y^\alpha)}{2}. \quad (2)
\]

Below we restrict our attention to regions of space–time in which there exist two points of intersection of the world line of the observer with the light cone. \( R \) and \( T \) as functions of the coordinates of the point \( y^\alpha \) observed are given implicitly by the following relations: \( \Phi(R, T, y^\alpha) = E[x^\alpha(T-R), y^\alpha] = 0 \), \( F(R, T, y^\alpha) = E[x^\alpha(T+R), y^\alpha] = 0 \).


The choice of the canonical parameter as the proper time of the observer is not necessary for the definition of chronometric \( R \) and \( T \). In exactly the same way it is possible to choose in a completely arbitrary manner a rule defining \( R \) and \( T \) as functions of \( s_1 \) and \( s_2 \) — the moments of transmission and reception of the signal.

In the region of space–time in question it is possible to choose as space–time coordinates \( R, T, \xi, \eta \), where the coordinates \( \xi, \eta \) are the "angles" on the surface \( R, T = \text{const} \). The choice of \( R, T(s_1, s_2) \) in form (1), (2) determines the components of the metric tensor

\[
g^{01} = 0, \quad g^{00} = -g^{11} = \frac{1}{2\Phi_R F_R} \Phi_\xi F_\xi. \quad (3)
\]

The coordinate transformation \( x^{\alpha}(x^\beta) \) occasioned by changing the choice of parameter and the law \( R, T(s_1, s_2) \) has the form

\[
R' = R'(R, T); T' = T'(R, T); \xi' = \xi'(R, T, \xi, \eta); \eta' = \eta'(R, T, \xi, \eta). \quad (4)
\]

Transformation (4) coincides with the transformation in the \( 1/V, 1/P \) dyad method. Here formulas (3) are invariant under the more restricted transformation

\[
R' = \frac{1}{2} [f(T + R) - f(T - R)], \quad T' = \frac{1}{2} [f(T + R) + f(T - R)],
\]

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