FOUR-FERMION QUARK DECAYS

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Four-fermion weak decays of free quarks are considered in the case of broken SU(4) symmetry. Beta-type decays of quarks into leptons are investigated, and \( t_c = 8.38 \times 10^{-15} \) sec is obtained for the charmed quark. Weak decays of quarks into baryons in the case of SU(4) symmetry are also investigated.

In this paper, we investigate weak four-fermion interactions in terms of the invariants of the group \( G = L \otimes SU(4) \), where \( L \) is the Lorentz group. In \([1, 3]\), a formal mathematical scheme of four-fermion weak interactions was constructed in the case of \( G' = L \otimes SU(3) \) symmetry. The results of these papers can also be generalized to the group \( G = L \otimes SU(4) \).

It is assumed that the Hamiltonian of the four-fermion weak interaction is invariant under transformations of the group \( G \), whose fundamental representation corresponds to the bispinor \( \Psi^A = \psi^A \). Here \( A \equiv (\alpha, I) \) contains two indices: \((\alpha = 1, 2, 3, 4)\), the spinor index of \( L \), and \((I = 1, 2, 3, 4)\), the unitary index of \( SU(4) \).

1. Beta Decay of Quarks

By analogy with the Hamiltonian of weak leptonic decays, we can construct the Hamiltonian of the \( \beta \) decay of quarks into leptons \([1]\):

\[
H = \sum_{j=1}^{5} \left( a^j_1 \left( q_i^j Q_{i}^j q_i^j \right) \left( \bar{\psi}_{6}^j \xi_{6}^j \right) + a^j_2 \left( q_i^j Q_{i}^j q_i^j \right) \left( \bar{\psi}_{6}^j \xi_{6}^j \right) \right),
\]

where \( Q^j = 1, \gamma_\alpha, \gamma_\beta, i\gamma_\alpha \gamma_\beta, \gamma_\alpha \gamma_\beta \) are Dirac matrices; \( a^j_{1,2} \) are ten arbitrary constants; \((\alpha, \beta, \gamma, \chi = 1, 2, 3, 4)\) are spinor indices; \((I, \bar{I} = 1, 2, 3, 4)\) are unitary indices; and \( \psi_{6}^j \) and \( \xi_{6}^j \) are the wave functions of the quark and lepton states, respectively.

If \( m_\beta = 0 \), then in (1.1) there remain only the vector \( a^j_{1,2} \) and pseudovector \( a^q_{1,2} \) interaction variants:

\[
H = a^j_1 \left[ q_i^j (1 + a_1 \gamma_0) q_i^j \right] \left[ \bar{\psi}_{6}^j (1 + \gamma_6) \xi_{6}^j \right] + a^j_2 \left[ q_i^j (1 + a_2 \gamma_6) q_i^j \right] \left[ \bar{\psi}_{6}^j (1 + \gamma_6) \xi_{6}^j \right],
\]

where \( a_1 \equiv -a_1^j/a^j_1, a_2 \equiv -a_1^j/a^j_2 \).

If we restrict ourselves to the case \( a_1 = a_2 = -\beta \), then we arrive at a quark–lepton interaction Hamiltonian of the form

\[
H_1 = \frac{G}{V^2} \left\{ J_{\alpha}^\mu \gamma_\alpha^\mu + \alpha (J_{\alpha}^I \gamma_I^\mu) \right\},
\]

\[
J_{\mu}^\alpha = \left[ q_i^\mu (1 - \gamma_\beta) q_i^\mu \right], J_{\alpha}^I = \left[ \bar{\psi}_{6}^j (1 + \gamma_\beta) \right] \xi_{6}^j;
\]

where \( G, \alpha, \beta \) are arbitrary constants, \( H_1 \) describes the scattering of quarks on leptons, and \( H_2 \) the decays of quarks into leptons.

On the basis of the requirement \( m_\mu > m_\tau > m_d, u \), the Hamiltonian which describes the quark decays can be written in the form

\[
H = G/V^2 \left\{ J_{\mu}^a \gamma_\mu^a + J_{\alpha}^a \gamma_\alpha^a + J_{\mu}^\alpha \gamma_\alpha^\mu + J_{\mu}^\alpha \gamma_\alpha^\mu + J_{\alpha}^I \gamma_\alpha^I + J_{\mu}^I \gamma_\mu^I \right\}.
\]

It is well known that the muon lepton charge \( L_\mu \) and the electron lepton charge \( L_e \) are conserved separately. Of course, the processes \( \mu^+ \rightarrow e^+ \), \( \mu^0 \rightarrow e^- e^- e^+ \), \( \mu^+ \rightarrow e^- e^- e^+ \), \( \gamma_\mu \rightarrow \gamma_\mu \), which


0038-5697/80/2308-0659$07.50 © 1981 Plenum Publishing Corporation
do violate the muon and electron lepton charges, are discussed in the literature [4, 5], but the latest experimental data indicate that these processes are suppressed compared with $\mu \rightarrow \nu_e e^+ e^-$ by $10^{-9}$ [6].

Taking into account the conservation of lepton charge and Cabibbo mixing of the quarks, we obtain from (1.4)

$$M_{12} = \frac{G}{\sqrt{2}} \cos \Theta_c \left( \bar{u}_\mu (1 - \beta \gamma_5) d \right) \left( \bar{\nu}_\tau (1 + \gamma_5) e \right); \quad M_{12} = \frac{G}{\sqrt{2}} \sin \Theta_c \left( \bar{u}_\mu (1 - \beta \gamma_5) s \right) \left( \bar{\nu}_\tau (1 + \gamma_5) e \right);$$

$$M_{34} = \frac{G}{\sqrt{2}} \cos \Theta_c \left( \bar{u}_\mu (1 - \beta \gamma_5) C \right) \left( \bar{\nu}_\tau (1 + \gamma_5) \nu \right); \quad M_{34} = \frac{G}{\sqrt{2}} \sin \Theta_c \left( \bar{u}_\mu (1 - \beta \gamma_5) C \right) \left( \bar{\nu}_\tau (1 + \gamma_5) \nu \right).$$

Here it is assumed that the coupling constant is $G = G_F = 10^{-5}/m^2$. $M_{ij}$ are the matrix elements of the corresponding processes.

As an example, we consider the decay $C \rightarrow S \mu^+ \nu_\mu$.

In the first perturbation order, the Feynman diagram has the form

$$\begin{array}{c}
\begin{array}{c}
\begin{array}{cc}
\overset{\mu^+}{\rightarrow}
\end{array}
\end{array}
\begin{array}{c}
\overset{q_2(E_q, \vec{q}_2)}{\rightarrow}
\end{array}
\begin{array}{c}
\overset{q_1(E_q, \vec{q}_1)}{\rightarrow}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\overset{C}{\rightarrow}
\end{array}
\begin{array}{c}
\overset{S}{\rightarrow}
\end{array}
\begin{array}{c}
\overset{\nu_\mu}{\rightarrow}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\overset{p_1(E_p, \vec{p}_1)}{\rightarrow}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\overset{p_2(E_p, \vec{p}_2)}{\rightarrow}
\end{array}
\end{array}
\end{array}
$$

where $p_1, p_2, q_1, q_2$ are the four-momenta of $C, S, \mu^+, \nu_\mu$, respectively.

The total decay probability can be calculated in accordance with

$$W = \frac{1}{(2\pi)^3} \int \frac{d^4(p_1 - p_2 - q_1 - q_2)}{2E_c} \frac{dp_2}{2E_2} \frac{dq_1}{2E_1} \frac{dq_2}{2E_2} |M_{34}|^2.$$

Making the necessary standard calculations, we obtain for the total decay probability

$$W = \frac{2G^2 \cos^2 \Theta_c}{(2\pi)^3} \left[ \frac{m_s^2}{2} \left( m_s - \frac{m_e}{2} \right)^2 \beta^2 + \left( m_c + \frac{m_s}{2} \right)^2 \right] \left[ m_e \sqrt{m_c^2 - m_s^2} + \frac{m_c m_s}{2} \right].$$

Here it is assumed that $m_c = m_s/2 = 1.6$ GeV, $m_s = m_s/2 = 0.6$ GeV, $\Theta_c = 15^\circ$, taking into account the two decay channels and three quark colors, we find that the lifetime of the $c$ quark is

$$\tau_c \approx 8.37 \times 10^{-15} \text{ sec}.$$

It is assumed here that the mass of the $c$ quark is $m_c = 2$ GeV, i.e., decays of quarks into baryons are energetically suppressed. If, in fact, the mass of the $c$ quark is above the threshold for production of a baryon-antibaryon pair, quark decays into baryons through the weak channel are possible.

2. Decays of Quarks into Baryons

The baryon states, as direct products of three quarks, are described by the wave function $\Psi_{ABC} \equiv \Psi_{\alpha \beta \gamma}$ and satisfy the Bargmann–Wigner equation

$$(i\gamma_\mu P_\mu + m_\alpha) \Psi_{\alpha} = (i\gamma_\mu P_\mu + m_\alpha) \Psi_{\beta} \Psi_{\gamma}^{\text{in}} = (i\gamma_\mu P_\mu + m_\gamma) \Psi_{\alpha} \Psi_{\beta}^{\text{in}} = 0.$$ (2.1)

By analogy with the ordinary four-fermion interaction, one can construct the Hamiltonian of weak decays of quarks into baryons: