which differs from the quasicyclic type in that dissimilar elements are on the principal diagonal. The properties of the determinant of this kind have not been studied, and formulas to evaluate them in the general case are not known. Hence, at this time the problem can be solved only if the elements on the main diagonal of the determinant (6) agree. A simple analysis of the determinant in (1) shows that such agreement will be observed if edges of just the type \( j^{(1)} \) are in the lattice. Such a picture is actually observed in all the problems solved (Fig. 3). Hence, the difference in principle between the solved and unsolved Ising problems is not the difference between two- and three-dimensional models or ordered and disordered systems but, more often, between problems about taking account of the second sphere for the rhombic (Fig. 3b) and square (Fig. 1) Ising lattice. The whole difficulty can apparently be concentrated in the following question: find at least one eigenvector of the matrix (6). The elements of the determinant \( A, B, ..., C, A_i \) can be considered numbers.

LITERATURE CITED

QUANTUM AND RELATIVISTIC VIRIAL INEQUALITIES

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The generalization of the virial theorem is discussed. The case where the potential energy is a sum of homogeneous functions of various degree is investigated. If the potential energy \( U \) is composed of a gravitational (or Coulomb) energy and an energy of the short-range repulsion of particles, then virial inequalities of the form \( 2K + \sum U < 0 \) are valid, where \( K \) is the kinetic energy. For classical systems of this type, but with a Hamiltonian relativistic in the momenta, the inequality \( 3N\Theta < |U| \) holds, where \( N \) is the number of particles in the system, \( \Theta = kT \), \( T \) is the temperature, and \( k \) is Boltzmann's constant.

The general virial theorem is known in two variants: the mechanical and the classical statistical. In the mechanical variant, it pertains to an average over an infinite time interval and asserts [1] that for spatially bounded systems

\[
\frac{\partial H}{\partial p_\alpha} = \frac{\partial H}{\partial q_\alpha},
\]

where, by definition,

\[
\overline{F} = \lim_{T \to \infty} \frac{1}{T} \int F(t) \, dt.
\]

From Eq. (1) we obtain, for a system of N particles:

\[ \sum_{k=1}^{3N} p_k \frac{\partial H}{\partial p_k} = \sum_{k=1}^{3N} q_k \frac{\partial H}{\partial q_k}, \]

(3)

where the expression on the right is, in Clausius's terminology, the average virial \( V \) of the system.

For a classical mechanical system the left-hand side of Eq. (3) is twice the kinetic energy \( K \), i.e.,

\[ 2\overline{K} = \overline{V}. \]

(4)

It is well known (see [2], for example) that for quantum systems relations of the form of Eqs. (1), (3), and (4) are valid for quantum-mechanical averages, i.e.,

\[ \langle p_k \overline{\frac{\partial H}{\partial q_k}} \rangle = \langle q_k \overline{\frac{\partial H}{\partial q_k}} \rangle, \]

(5)

\[ \sum_{k=1}^{3N} \langle p_k \overline{\frac{\partial H}{\partial q_k}} \rangle = \sum_{k=1}^{3N} \langle q_k \overline{\frac{\partial H}{\partial q_k}} \rangle. \]

(6)

For systems having a nonrelativistic kinetic energy \( K = \sum_{i=1}^{3N} \frac{1}{2m_i} p_i^2 \) it is obvious that

\[ 2 \langle K \rangle = \langle V \rangle. \]

(7)

In the classical statistical variant, the virial theorem asserts [1] that for spatially bounded systems the classical phase averages satisfy the relations:

\[ p_k \frac{\partial H}{\partial p_k} = q_k \frac{\partial H}{\partial q_k} = \Theta \]

(8)

or

\[ \sum_{k=1}^{3N} p_k \frac{\partial H}{\partial p_k} = 3N\Theta, \quad \sum_{k=1}^{3N} q_k \frac{\partial H}{\partial q_k} = \overline{V} = 3N\Theta, \]

(9)

where \( \Theta = kT \). If the momentum part of the Hamiltonian has the form of a nonrelativistic kinetic energy, then from Eq. (9) we get

\[ 2\overline{K} = 3N\Theta. \]

(10)

From Eqs. (10) and (9), just as in the cases of Eqs. (4) and (7), we have:

\[ 2\overline{K} = \overline{V}. \]

(11)

Equations (8)-(10) are valid only in the case of classical statistics, whereas Eq. (11) coincides with the quantum relation (7) and is evidently valid in quantum statistics as well.

Equations (1), (3), (5), (6), (8), and (9) are derived under a general assumption of spatial boundedness, for arbitrary Hamiltonians \( H \). Among these, the momentum part of the Hamiltonian \( K \) can be taken in the form

\[ K = \sum_{k=1}^{N} m_k c^2 \left( \sqrt{1 + \frac{p_k^2}{m_k^2 c^2}} - 1 \right), \]

(12)

i.e., as the kinetic energy of a relativistic gas. In this way, Eqs. (1), (3), (5), and (6) are also valid for relativistic systems, in which, however, the interaction is taken into account only through the introduction of a potential energy \( U \) (i.e., radiation and particle creation are not accounted for), and the spin of the particles is ignored. Then Eqs. (4) and (7) do not follow from Eqs. (3) and (6), respectively, since in general

\[ \sum_{k=1}^{3N} p_k \frac{\partial H}{\partial p_k} \neq 2K. \]