calculations were carried out for $\gamma_1 = 1.55; \ z_0 = 0.11 (\phi_2 = 0); \ z = 0.15 (\phi_2 = -0.33); \ h\omega = 0.117 \text{ eV}$.

These calculations thus gave a general expression for the drag current of holes, which allows for the nonparabolicity of the light-hole band of a narrow-gap semiconductor, and the parameters necessary to obtain the temperature dependences of the drag current were found: these are the quantity representing the light-hole band nonparabolicity and the elementary tensors. A subsequent comparison of the calculated and experimental dependences $j_h(T)$ can be used to estimate the contribution, in a wide temperature range, of various carrier scattering mechanisms for narrow-gap semiconductors of cubic symmetry such as p-type $\text{Hg}_1-x\text{Cd}_x\text{Te}$ solid solutions.

LITERATURE CITED


EQUATIONS WITH A SELF-CONSISTENT FIELD IN QUANTUM-STATISTICAL THEORY OF A CRYSTAL

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A system of equations with a self-consistent field is derived for the density of particles in the quantum case. This system has periodic solutions for a crystal. Equations for the Fourier coefficients are deduced from these solutions and are used to find equations containing only the Fourier coefficients of the density. A method is also given for solving the initial system by an expansion in powers of the Planck constant $\hbar$; the terms proportional to $\hbar^2$ and $\hbar^2$ are calculated.

In investigations of systems containing large numbers of particles an effective approach is provided by the self-consistent field method. Application of this method in the statistical theory of solids was started by A. A. Vlasov; this approach has been developed significantly in the last decade. However, most authors have dealt with the classical case. Among work on quantum-statistical theory of a crystal based on the self-consistent method we shall mention [1-3].

In the present paper the Vlasov equation for a crystal is generalized to the quantum case employing a different approach to the problem than that used in the cited papers; equations are obtained which give directly a periodic density. Allowance is also made.

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for the exchange effects ignored in [1-3].

We shall first consider the dependence of the steady-state density $\rho(r)$ on the potential energy $U(r)$ of noninteracting particles in an external field. If $n(\varepsilon_K)$ is the number of particles in a state of energy $\varepsilon_K$ with a wave function $\psi_K$, the density is

$$\rho(r) = \sum_\varepsilon n(\varepsilon) |\psi_\varepsilon|^2 = \frac{1}{(2\pi)^3} \int_\infty \exp\left(-\frac{i}{\hbar} pr\right) n(\hat{H}) \exp\left(\frac{i}{\hbar} pr\right) dp. \tag{1}$$

In deriving the last relationship we have allowed for the fact that $n(\varepsilon) \psi_\varepsilon = n(\hat{H}) \psi_\varepsilon$, where $\hat{H}$ is the Hamiltonian of a particle, and have applied the Fourier transformation to the functions $\psi_K$ we have followed this by utilizing the completeness of the system of these functions. The result of the action of the operator $n(\hat{H})$ can be represented as

$$n(\hat{H}) \exp\left(\frac{i}{\hbar} pr\right) = \frac{1}{2\pi i} \int_C ds n(s) \frac{1}{s - \hat{H}} \exp\left(\frac{i}{\hbar} pr\right). \tag{2}$$

The contour $C$ should enclose points $s = \varepsilon_K$ whereas singular points $n(s)$ should lie outside $C$. As $C$ we can take a contour starting from $+\infty$ in the upper-plane $s$ parallel to the real axis, bypassing anticlockwise the point $s = \min \varepsilon_K$ and returning to $+\infty$.

Substituting Eq. (2) in Eq. (1), we obtain

$$\rho(r) = \frac{1}{(2\pi)^3 i \hbar^3} \int_\infty \int_C ds u(r, p, s) n(s), \tag{3}$$

where $u = \exp(-i/\hbar r)(s - \hat{H})^{-1} \exp(i/\hbar r)$. Taking as $n(s)$ a Fermi or a Bose distribution, we obtain

$$\rho(r) = \frac{g}{(2\pi)^3 i \hbar^3} \int_\infty \int_C \frac{u(r, p, s)}{\exp[(s - \mu)/\theta] + 1} ds, \tag{4}$$

where $g$ is the multiplicity of degeneracy of the levels associated with the presence of spin; $\theta$ is the temperature; here and later the upper sign refers to the Fermi statistics and the lower to the Bose statistics. It should be noted that singularities $u(r, p, s)$ lie within the contour $C$.

The definition of $u$ allows us to derive easily the equation which this function satisfies:

$$\frac{\hbar^2}{2m} \Delta u + \frac{i\hbar}{m} p \nabla u + \left(s - \frac{p^2}{2m} - U\right) u = 0. \tag{5}$$

Equation (4) together with Eq. (5) give the dependence of the density $\rho(r)$ on the potential $U(r)$. Assuming that each particle moves in a self-consistent field, we can describe the potential $U(r)$ by

$$U(r) = \int K(|r - r'|) \rho(r') dr', \tag{6}$$

where $K(|r - r'|)$ is the potential energy of the interaction between particles (pair central forces are assumed).

Equations (4)-(6) represent a closed system with a self-consistent field. The crystal state corresponds to periodic solutions of this system. We shall seek such solutions in the form of Fourier series:

$$\rho(r) = \sum_{l, m, n} a_\alpha e^{i\alpha r}; u(r, p, s) = \sum_{l, m, n} b_\alpha(p, s) e^{i\alpha r}, \tag{7}$$

where $\alpha = la_1 + ma_2 + na_3$, the index $\alpha$ denoting the set of numbers $l$, $m$, and $n$.

Substitution in Eq. (6) gives

$$U(r) = \sum_{l, m, n} a_\alpha \sigma(\alpha) e^{i\alpha r} dr, \tag{8}$$

where $\alpha = l\alpha + m\alpha + n\alpha$, the index $\alpha$ denoting the set of numbers $l$, $m$, and $n$. 

415