It follows from (17) and (18) that with the integration of Eq. (13) the method of successive approximations leads to the generalized virial expansion [3].

LITERATURE CITED

NUMERICAL DENSITY OF $^3$He IN COEXISTING PHASES OF THE SYSTEM
CO$_2$--$^3$He

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The method of slow-neutron transmission is used to investigate the behavior of the numerical density of $^3$He in the binary solution 2.46\% $^3$He + 97.54\% CO$_2$ on the liquid–vapor boundary curve. Data on the numerical density are analyzed within the framework of the expanded theory of scale transformations.

Theoretical analysis of the behavior of binary solutions close to the liquid–vapor critical point became possible with the introduction of the hypothesis that critical phenomena are isomorphic [1, 2] and with the derivation of the isomorphic equation of state of binary solutions [3]. Verification of the isomorphic equation of state of binary solutions requires combined data on the specific heat, the temperature and pressure dependence of the mixture density, and the temperature dependence of the numerical density of the components of solution on the liquid–vapor boundary curve in the vicinity of the critical point. To date, there have been few works on the density of binary solutions at the liquid–vapor boundary curve and practically no data on the behavior of the numerical density of the solution components on this curve.

With a definite choice of the binary-mixture components, the problem of finding the numerical density of mixture components on the boundary curve may be solved using the method of slow-neutron transmission.

THEORY OF THE METHOD

As is known [4], a height distribution of both the density $\rho = \rho(h, T)$ and the numerical density $N_1(h, T), N_2(h, T)$ of the binary-mixture components arises close to the critical point of vapor formation of binary solutions, on account of the anomalous increase in compressibility of the material in a gravitational field. The numerical density of the components on the boundary curve may be obtained as a result of investigating the dependences $N_1(h), N_2(h)$ at different temperatures $T \leq T_{cr}$, where $T_{cr}$ is the critical temperature of the mixture.

In the method of slow-neutron transmission, the height dependence of the numerical density of binary-mixture components at various temperatures may be found as a result of solving a system of equations in which each equation corresponds to a definite stage of the neutron experiment

\[
\begin{align*}
\rho &= \rho_0 \exp(-\Sigma l); \\
\rho_1 &= \rho_1 \exp(-\tilde{N}_1 \sigma_1 x - \Sigma l); \\
\rho_2 &= \rho_2 \exp(-\tilde{N}_2 \sigma_2 x - \Sigma l); \\
\rho(h, T) &= \rho_0 \exp(-N_1(h, T) \sigma_1 x - N_2(h, T) \sigma_2 x - \Sigma l),
\end{align*}
\]

where $I$ is the macroscopic cross section of neutron interaction with the walls of the sample of thickness $l$; $\sigma_1$, $\sigma_2$ are the cross sections of neutron interaction correspondingly with the molecules of the first and second components of the mixture; $x$ is the sample thickness; $i_0^0 = I_0^0/M_0^0$, $i^0 = I^0/M^0$ are normalized (to the monitor reading $M$) intensities of the neutron beam before and after the container, with measurements in an empty sample; $i_1^0$, $i^1$ are the same quantities with measurements in a sample filled with the first component, which is uniformly distributed over the sample volume at a mean numerical density $\bar{N}_1$; $i_2^0$, $i^2$ are the same quantities obtained with measurements in a binary mixture with $T \gg T_{cr}$, when the mixture density is uniformly distributed over the sample volume; $i_o$, $i(h, T)$ are the same quantities obtained when a narrow neutron beam passes through the sample at height $h$, measured from the level of the phase interface for temperatures $T \ll T_{cr}$. The mean values of the numerical density $\bar{N}_1$ and $\bar{N}_2$ are determined experimentally with known values of the sample volume $V$, masses $m_1$ and $m_2$ of the mixture components in the sample, and are related to the mean density of the mixture in the sample (filling density) by the expression

$$\bar{\rho} = \frac{m_1 + m_2}{V} = \frac{1}{N_A} (\bar{N}_1 \sigma_1 + \bar{N}_2 \sigma_2).$$

where $N_A$ is Avogadro's number; $\mu_1$, $\mu_2$ are the molecular weights of the components.

The solution of Eq. (1) takes the form

$$N_1(h, T) = N_1 \left[ \frac{\ln \left( \frac{i(h, T)i_0^0}{i_0^0} \right)}{i_1^0 \ln \left( \frac{i(h, T)i_0^0}{i_0^0} \right)} + \frac{\bar{N}_1(h, T)}{\bar{N}_1} \ln \left( \frac{i^1}{i^0} \right) \ln \left( \frac{i^1}{i^0} \right) \right].$$

(3)

In the case where $\sigma_1 \gg \sigma_2$ and $\bar{N}_1 \sigma_1 \gg \bar{N}_2 \sigma_2$, the second term in the numerator of Eq. (3) is small in comparison with the first term, since $i_1^0/i_0^0 \approx i_2^0/i_0^0$. In this case, the method of slow-neutron transmission allows the height dependence of the numerical density of the component with the larger neutron cross section to be investigated. This case may be realized for the mixture $CO_2$--$^3$He ($\sigma_{3He} \approx 5400$ b; $\sigma_{CO_2} \approx 13$ b [5]).

For the case where $\bar{N}_1 \sigma_1 \approx \bar{N}_2 \sigma_2$, the second term of the numerator in Eq. (3) is not a small quantity and must be taken into account. If the correction term in Eq. (3) is to be correctly taken into account, an additional experiment must be formulated, allowing the distribution of the binary-solution density to be investigated at various temperatures.

In the case where $\sigma_1 \approx \sigma_2$, the method of slow-neutron transmission does not allow the height distribution of the density of the individual solution components to be investigated, but allows the distribution of the binary-mixture density over the height to be correctly investigated. The strong dependence which exists between the total neutron cross section and the isotopic composition of the material means that it is possible to choose the pair of binary-solution components for which $\sigma_1 \approx \sigma_2$; with isotopic substitution, $\sigma_1 \gg \sigma_2$, which opens up the possibility of using the method of slow-neutron transmission to investigate the temperature and field dependence of both the density of the solution and the density of its individual components.

**EXPERIMENTAL**

On the WR-M reactor of the Institute of Nuclear Research, Academy of Sciences of the Ukrainian SSR, an experiment was performed to investigate the numerical density of $^3$He in coexisting phases of the system 2.46% $^3$He + 97.54% CO$_2$ (mole. %). A diagram of the apparatus and the experimental geometry may be found in [6]. An aluminum sample with an internal volume of 200 x 50 x 10 mm was filled with the experimental solution and than placed in a thermostated container fitter to a device permitting vertical motion. This device allowed the sample to be introduced into the neutron beam and moved over the height with a minimal step of 0.1 mm. The sample temperature was maintained constant with an error of 0.001°K. The sample temperature was measured using a platinum resistance thermometer and an R-363 potentiometer. The temperature gradient over the height of the sample (200 mm) was recorded by means of a fifteen-junction copper-Constantan thermocouple and was more than 10⁻³°K/mm. The intensity of the neutron beam passing through the sample was ~$1 \times 10^6$ min⁻¹ which allows the transmission of the sample to be determined with a statistical error of ~0.1%. 

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