On the basis of a general connection between a quantum and a semiclassical description, a modified system of dynamic equations of a semiclassical theory is proposed which takes into account quantum and statistical properties of interacting subsystems more fully than the traditional theory. Modified equations of semiclassical electrodynamics and semiclassical interparticle interaction are given. The possibility of macroscopic manifestation of quantum and statistical corrections are discussed.

INTRODUCTION

It is known that semiclassical theories, which are rather simple in formulation and in the mathematical technique used, reflect many of the features of the exact quantum description of interacting subsystems [1-3]. It is clear that this feature is a consequence of the deep connection between the semiclassical and the full quantum description which, apparently, enables one to construct semiclassical representations of the quantum theory [4]. A semiclassical representation establishes a correspondence between results obtained using a semiclassical approach and a well-defined approximation to the quantum theory, and allows one to find the solution to a quantum problem, starting from the solution of the corresponding problem found in the semiclassical approximation.

Although a semiclassical representation, in principle, solves the problem of calculating quantum and statistical corrections to the results of the semiclassical approximation and of clarifying their role in specific physical situations, direct use of relationships of the semiclassical representation method for realistic models leads to significant computational difficulties. Thus, in the operator version of the semiclassical representation method, the problem of determining the magnitude and the role of these corrections is reduced to solution of a nonlinear Schrödinger equation. Here, secular terms appear immediately, and it is very difficult to construct the regular perturbation theory in the general case, although some interesting results have been obtained using this approach [5]. In the statistical version of the semiclassical representation method the same problem is reduced to solution of the multidimensional Fokker-Plank equation with nonlinear friction and diffusion coefficients. In spite of the fact that for simple models this approach allows one to solve the initial quantum problem, starting from the results of a semiclassical analysis (e.g., in the quantum theory of lasers [6]), in a general case this approach is very laborious.
Quantum corrections to the semiclassical approximation seem to be negligible, especially in those situations where one of the subsystems is quasiclassical (heavy particles, strong field). However, the fact that equations of motion of the semiclassical description are significantly nonlinear, i.e., small perturbations can significantly change the result of the evolution of the subsystem studied [7], forces one to exercise more caution towards the question of the possible effect of quantum and statistical corrections on the behavior of a quasiclassical subsystem interacting with the quantum surrounding. In spite of its formal complexity, the theory of nonlinear differential equations provides a convenient tool for the study of this question.

The present work is devoted to the construction of such a semiclassical theory, whose equations, as much as possible, reflect the fact that the initial problem about the subsystem interaction is a quantum theoretical problem. Although at first sight this formulation of the problem seems paradoxical, it will be shown later on that it is sufficiently real and leads to the appearance of additional terms, understandable from a physical point of view in new equations of motion of the semiclassical approximation. Construction of modified equations of motion is based only on certain requirements to the structure of the main operators of the semiclassical theory—these are evolution operators and Hamiltonians. In essence, "semiclassical character" is required only in the structure of equations of motion: the differential equation for c-functions, which are dynamic variables for the subsystem studied, and the Schrödinger equation for the surrounding interacting with the subsystem. The Hamiltonian in this equation should depend on both the corresponding operators of the surrounding and on dynamic variables, which are solutions of the equations indicated above.

Derivation of the new equations of motion of the semiclassical theory allowed us to construct a modified semiclassical representation of the evolution operator, which connects the semiclassical theory with the complete quantum description of interacting subsystems.

The treatment is illustrated using as an example the interaction of a single mode field with a quantum system and interaction between internal and external motion of interacting particles. Conditions for macroscopic manifestation of quantum and statistical corrections are discussed.

In order to simplify the derivation, the interaction between closed subsystems is considered. Using the results obtained in application of the semiclassical representation method to the theory of open systems [6], the proposed approach can be easily reformulated for this particular case, which is more interesting for applications.

1. The Method of Semiclassical Representation of Quantum Theory

We shall now give a brief description of the semiclassical representation method [4, 8, 9], whose analogy will be used to construct the modified semiclassical representation, which leads to equations of motion taking into account quantum and statistical corrections semiclassically.

We shall first introduce the main definitions. Subsystems whose evolution we wish to describe using equations of motion for c-functions will be called "classical," and the surrounding interacting with the subsystem will be called "quantum subsystem." We shall restrict ourselves to such classical subsystems whose behavior can be conveniently described using the language of canonically conjugate variables which satisfy canonical commutation relations, e.g., $[\hat{p}, \hat{q}] = -i\hbar$ or $[\hat{a}, \hat{a}^+] = 1$.

Further, the general treatment will be performed assuming that the quantum subsystem is described by the Hamiltonian $H_1(x)$, where $x$ denotes the required set of operators, the Hamiltonian of the classical subsystem is $H_2(\hat{a}^\pm)$, where $\hat{a}^+$ and $\hat{a}^-$ are creation and annihilation operators, respectively, which satisfy Bose commutation relations: $[\hat{a}, \hat{a}^+] = 1$ and $H_{12}(\hat{a}^\pm, x)$ is the interaction Hamiltonian between subsystems. The Hamiltonian $H$ for the quantum problem can now be written in the following form:

$$H = H_1(x) + H_2(\hat{a}^\pm) + H_{12}(\hat{a}^\pm, x).$$

We will assume that at time $t_0$ the interaction between subsystems is "switched on," and the initial density matrix of the system $\rho_0$ is given by $\rho_0 = \rho T$, where $\rho$ is the initial density matrix of the classical subsystem and $T$ is that of the quantum subsystem. The system evolution operator $S$ satisfies the following Schrödinger equation: