LIMITING EQUILIBRIUM OF LAMINAS AND SHELLS
OF ROTATION OF A COMPRESSIBLE MATERIAL

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Problems concerning the dependence of the yield condition on the average stress were considered in [6, 7, 8]. The limiting equilibrium of laminar structures made of an ideally plastic material, for which the yield condition depends on the average stress, was investigated in [3].

We shall consider the limiting equilibrium of solid laminas and shells of rotation made of the material investigated in [3].

§1. The condition of limiting equilibrium of ideally plastic isotropic media with a linear dependence of the limiting tangential force \( \tau \) on the normal pressure \( \sigma_n \) acting on an elementary area with normal \( n \), is written in the form [5]

\[
\max \{ |\tau_n| - \sigma_n \tan \phi \} = k,
\]

where \( \phi \) is the angle of internal friction; \( k \) is the coupling coefficient.

In the space of the principal stresses \( \sigma_i \) (\( i = 1, 2, 3 \)) condition (1.1) assumes the form [5]

\[
\frac{\sigma_i}{\sigma_j} + \frac{\sigma_j}{\sigma_i} \leq 1 \quad (i \neq j; i, j = 1, 2, 3).
\]

Here, the transient resistances as a result of tension and compression are determined by the expressions

\[
\sigma_r = \frac{2H \sin \phi}{1 - \sin \phi}; \quad \sigma_s = -\frac{2H \sin \phi}{1 + \sin \phi} \quad (\sigma_r > 0; \sigma_s < 0);
\]

the quantity \( H = k \cot \phi \) defines the transient resistance of the omnidirectional equilibrium tension.

We shall assume that the symmetrically stressed solid shell of rotation with thickness \( h \) is made of an isotropic ideally plastic material subjected to the yield condition (1.2). The corresponding yield hexagon is drawn in Fig. 1, where \( \sigma_1 \) and \( \sigma_2 \) are the principal stresses in the shell; \( \sigma_3 = 0 \).

In order to construct the bounded surface in the space of the generalized forces \( N_i; M_i \) (\( i = 1, 2 \)) the scheme is used which is given in [2]. When considering the plane \( e_1 e_2 \) (Fig. 2) we shall use the Kirchhoff - Love hypotheses

\[
e_1 = e_{10} + \kappa_1 z; \quad e_2 = e_{20} + \kappa_2 z; \quad \left( -\frac{h}{2} \leq z \leq \frac{h}{2} \right),
\]

where \( e_{10} \) and \( \kappa_1 \) (\( i = 1, 2 \)) are the principal rates of extensions and changes of curvature of the coordinates of the shell surface.

According to the associated law of flow, each of the six sectors generalized in Fig. 2 corresponds to an angle of the yield hexagon shown in Fig. 1.

If a point of the plane \( e_1 e_2 \) lies on one of the lines of separation, for example \( 0aF \) (where \( 0aF \) is the perpendicular dropped from the origin of the coordinates to the side \( AF \) of the yield hexagon), then the stressed state at this point corresponds to the direction of the condition of plasticity, to which this line of separation is orthogonal, i.e., \( AF \).
It follows from condition (1.4) that the distribution of deformations over the thickness of the shell in the plane $\varepsilon_1\varepsilon_2$ is represented by the intercept of the straight line KL, the boundary of which corresponds to the values $z = \pm h/2$.

We introduce the notation:

\begin{align*}
    h_p &= -\frac{\varepsilon_{10}}{\chi_1}; & h_r &= -\frac{\varepsilon_{30}}{\chi_2}; \\
    h_q &= -\frac{\varepsilon_{10} - \gamma\varepsilon_{20}}{\chi_1 - \gamma\chi_2}; & h_s &= -\frac{\gamma\varepsilon_{10} - \varepsilon_{30}}{\gamma\chi_1 - \chi_2}.
\end{align*}

Here $\gamma = \sigma_r/\sigma_s$; $p$, $q$, $r$, $s$ are the dimensionless coordinates of the layers of the shell in which the deformed state lies on the radials $0ef$, $0cd$, $ode$, and $0af$.

By considering the different positions of the straight line KL in the plane $\varepsilon_1\varepsilon_2$, we obtain analytical expressions for the generalized forces.

In the case when the points $K_1$ and $L_1$ correspond to the values $z = h/2$ and $z = -h/2$, the parametric description of part of the yield hypersurface assumes the form

\begin{align*}
    n_1(p) &= \sigma_r\left(\frac{1}{2} - p\right) + \sigma_s\left(q + \frac{1}{2}\right); & n_1^{(q)} &= \sigma_r\left(r - q\right); \\
    m_1(p) &= \sigma_r\left(\frac{1}{2} - 2p^2\right) + \sigma_s\left(2q^2 - \frac{1}{2}\right); & m_1^{(q)} &= 2\sigma_r\left(r^2 - q^2\right) \\
    \left(\frac{1}{2} \gg r \gg p \gg q \gg -\frac{1}{2}\right); \\
    m_1^{(q)} &= \sigma_r\left(p + \frac{1}{2}\right) + \sigma_s\left(\frac{1}{2} - q\right); & n_1^{(q)} &= \sigma_r\left(q + \frac{1}{2}\right) + \sigma_s\left(\frac{1}{2} - r\right); \\
    m_1^{(q)} &= \sigma_r\left(2p^2 - \frac{1}{2}\right) + \sigma_s\left(\frac{1}{2} - 2q^2\right); & m_1^{(q)} &= \sigma_r\left(2q^2 - \frac{1}{2}\right) + \sigma_s\left(\frac{1}{2} - 2r^2\right) \\
    \left(\frac{1}{2} \gg q \gg p \gg -\frac{1}{2}\right); \\
    m_1^{(r)} &= \sigma_r\left(p - s\right); & n_1^{(r)} &= \sigma_r\left(\frac{1}{2} - r\right) + \sigma_s\left(s + \frac{1}{2}\right); \\
    m_1^{(r)} &= 2\sigma_r\left(p^2 - s^2\right); & m_1^{(r)} &= \sigma_r\left(1/2 - 2s^2\right) + \sigma_s\left(2s^2 - \frac{1}{2}\right) \\
    \left(\frac{1}{2} \gg p \gg q \gg s \gg -\frac{1}{2}\right); \\
    n_1^{(s)} &= \sigma_r\left(\frac{1}{2} - s\right) + \sigma_s\left(p + \frac{1}{2}\right); & n_1^{(s)} &= \sigma_r\left(\frac{1}{2} - r\right) + \sigma_s\left(s + \frac{1}{2}\right); \\
    m_1^{(s)} &= \sigma_r\left(\frac{1}{2} - s\right) + \sigma_s\left(p + \frac{1}{2}\right); & m_1^{(s)} &= \sigma_r\left(\frac{1}{2} - r\right) + \sigma_s\left(s + \frac{1}{2}\right); \\
    \left(\frac{1}{2} \gg r \gg p \gg q \gg -\frac{1}{2}\right); \\
    n_1^{(q)} &= \sigma_r\left(\frac{1}{2} - q\right); & n_1^{(q)} &= \sigma_r\left(r - q\right); \\
    m_1^{(q)} &= \sigma_r\left(\frac{1}{2} - 2q^2\right) + \sigma_s\left(\frac{1}{2} - 2r^2\right). \\
\end{align*}