METHOD OF CORRELATION OPERATORS IN THE THEORY OF A SYSTEM
OF PARTICLES WITH STRONG INTERACTIONS

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The method of correlation operator in the theory of many-particle systems is
generalized to the case of finite temperature. A similarity transformation of
the density matrix is performed with the help of the correlation operator,
which does not change the value of the partition function. A method of calcu-
ling the transformed partition function with the help of a finite translation
operator is given. A general system of coupled equations is obtained from
which the matrix elements of correlation operators of increasing order can be
found.

The problem of calculating the properties of particles with strong interactions has long
occupied the attention of theorists. Among the many methods developed up to now there is the
method of Green's functions, the variational method, and the correlation operator method (at
zero temperature). The last in a sense combines the advantages of the first two methods.
The correlation operator method, like the Green's function method, gives a detailed physical
picture of the phenomena and also can be written in terms of diagrams, but like the varia-
tional method it can be extended to the range of strong interactions where the coupling con-
stant is not small [1, 2]. We develop and generalize the correlation operator method to the
case of nonzero temperature with the help of the methods of functional differentiation and
integration.

We consider a system of interacting fermions at a temperature \( T \equiv \beta^{-1} \). The partition
function of the system is

\[
Z = \text{Sp} \exp(-\beta H),
\]

where

\[
\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}},
\]

\[
\hat{H}_0 = \sum_{ps} (\epsilon(p) - \mu) \hat{a}_{ps}^+ \hat{a}_{ps};
\]

\[
\hat{H}_{\text{int}} = \frac{1}{2} \sum_{p, p_0, q, q_0} V(q) \hat{a}_{p_0 q_0}^+ \hat{a}_{p q} \hat{a}_{p_0 q} \hat{a}_{p q_0} + \text{h.c.};
\]

and \( \hat{a}_{ps}^+ \) and \( \hat{a}_{ps} \) are the creation and annihilation operators of particles with momentum \( p \)
and spin \( s \). Below we will omit the spin subscript, combining spin and momentum in a single
subscript.

We perform a similarity transformation on the density matrix:
\[ \exp(-\hat{S}) \exp(-\beta \hat{H}) \exp(\hat{S}). \]

It is easy to see that the partition function is unchanged under such a transformation. Operator \( \hat{S} \) is called the correlation operator and can be represented in the form [1]:

\[ \hat{S} = \sum_{n=1}^{N} \hat{S}_n = \sum_{n=1}^{N} \frac{1}{(n!)^2} \sum_{\lambda_1 \ldots \lambda_n \nu_1 \ldots \nu_n} \langle \gamma_1 \ldots \gamma_n | \hat{S} | \nu_1 \ldots \nu_n \rangle \alpha_{\lambda_1}^+ \ldots \alpha_{\lambda_n}^+ \alpha_{\nu_1} \ldots \alpha_{\nu_n}, \]

where the indices \( \lambda \) and \( \nu \) are defined such that

\[ \varepsilon_{\lambda} \equiv \varepsilon (p_{\lambda}) \geq \nu; \]
\[ \varepsilon_{\nu} \equiv \varepsilon (p_{\nu}) < \nu. \]

A denotes an antisymmetrization with respect to variables \( \nu_1, \ldots, \nu_n \) of the state \( |\nu_1, \ldots, \nu_n\rangle \).

We write (5) as a single exponential:

\[ \exp(-\hat{S}) \cdot \exp(-\beta \hat{H}) \cdot \exp(\hat{S}) = \exp(-\beta \hat{H}_{\text{eff}}), \]

where

\[ \hat{H}_{\text{eff}} = \exp(-\hat{S}) \hat{H} \exp(\hat{S}) = \hat{H} + \sum_{n=1}^{4} \hat{D}_n = \hat{H} + \hat{R}, \]

and \( \hat{D}_n = \frac{1}{n!} \left[ ... \left[ \hat{H}, \hat{S}, ... \hat{S} \right] \right] \) is the commutator of \( n \)-th order.

The partition function becomes

\[ Z = \text{Sp} \exp(-\beta \hat{H}_{\text{eff}}). \]

Obviously, one can also consider the partition function after a dual transformation of the form

\[ Z = \text{Sp} \exp(-\hat{S}^+) \exp(-\hat{S}) \exp(-\beta \hat{H}) \exp(\hat{S}) \exp(\hat{S}^+). \]

It can be shown that from the condition that the thermodynamic potential be a minimum, follows a chain of coupled equations determining the matrix elements of the correlation operator \( \hat{S} \):

\[ \frac{1}{Z} \text{Sp} (a_{\lambda_1}^+ \ldots a_{\lambda_n}^+ a_{\nu_1} \ldots a_{\nu_n} \exp(-\beta \hat{H}_{\text{eff}})) = 0, \quad n = 1, 2 ... N. \]

We introduce an external source of pairs of particles of the form

\[ \hat{U} = \sum_{n=1}^{N} U_n (\gamma_1 \ldots \gamma_n, \lambda_1 \ldots \lambda_n) a_{\gamma_1}^+ \ldots a_{\gamma_n}^+ a_{\lambda_1} \ldots a_{\lambda_n}. \]

Then the system of equations (12) can be written in the form

\[ \frac{\delta}{\delta U_n (\gamma_1 \ldots \gamma_n, \lambda_1 \ldots \lambda_n)} \ln Z_n = 0, \quad n = 1, 2 ... N, \]

where

\[ Z_n = \text{Sp} (\exp(\hat{U}) \cdot \exp(-\beta \hat{H}_{\text{eff}})). \]

After differentiation, the external source is set equal to zero. Combining the exponents in (15) and omitting higher-order commutators, the partition function can be written in the form

\[ Z_n = \text{Sp} (-\beta \hat{H} - \beta \hat{Q}), \]